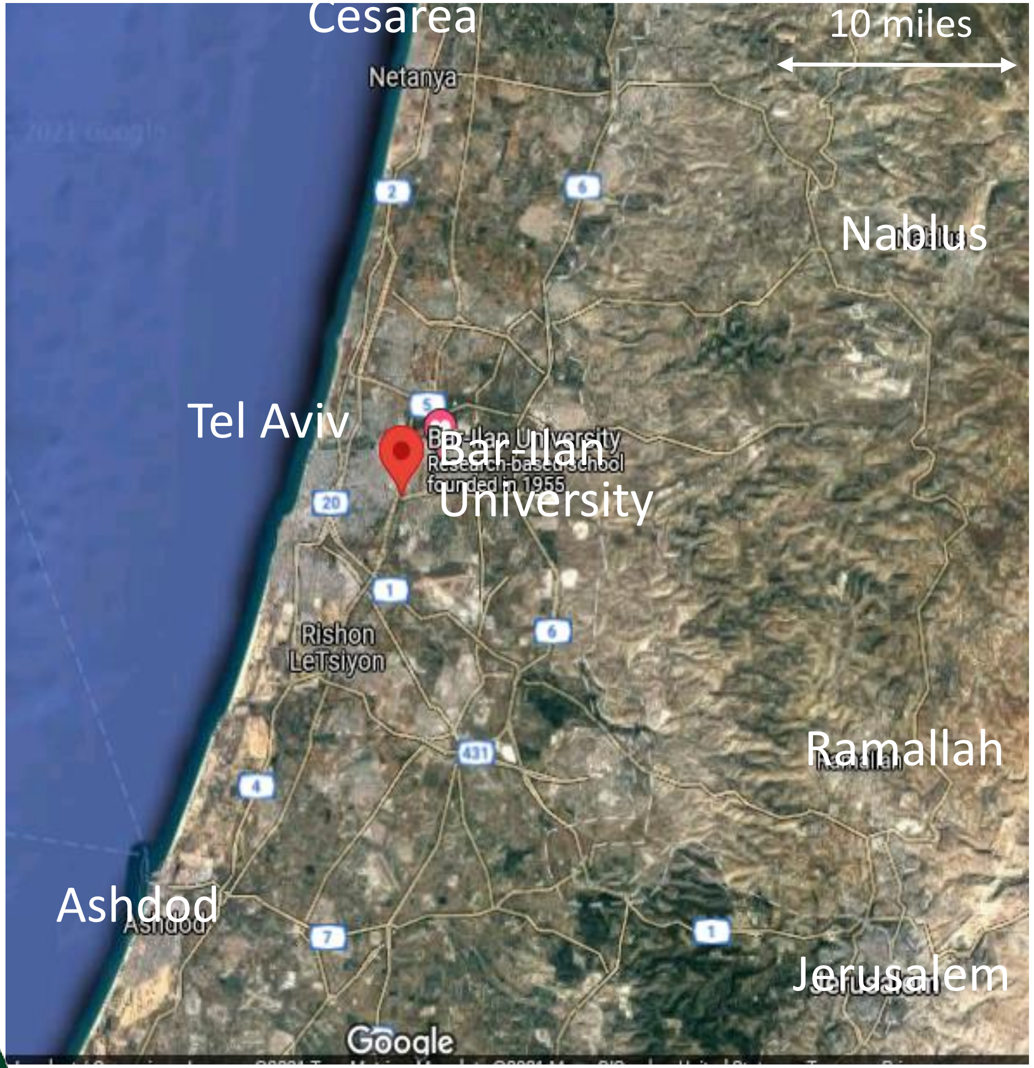
Parametrically Driven Workshop, Zurich, 13/01/2023

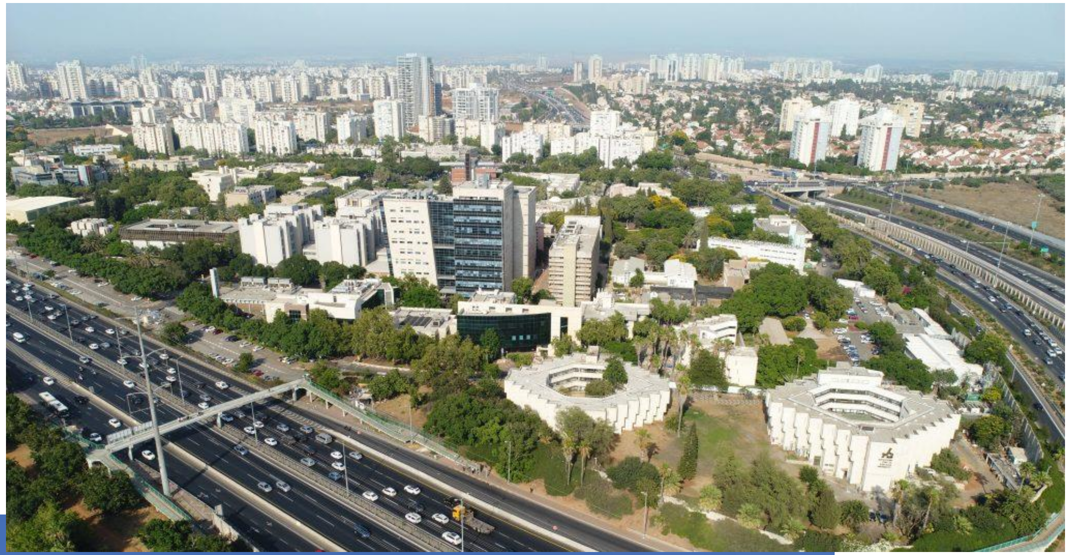
Parametric oscillators: Ising machines, prethermalization, and squeezing

Emanuele Dalla Torre

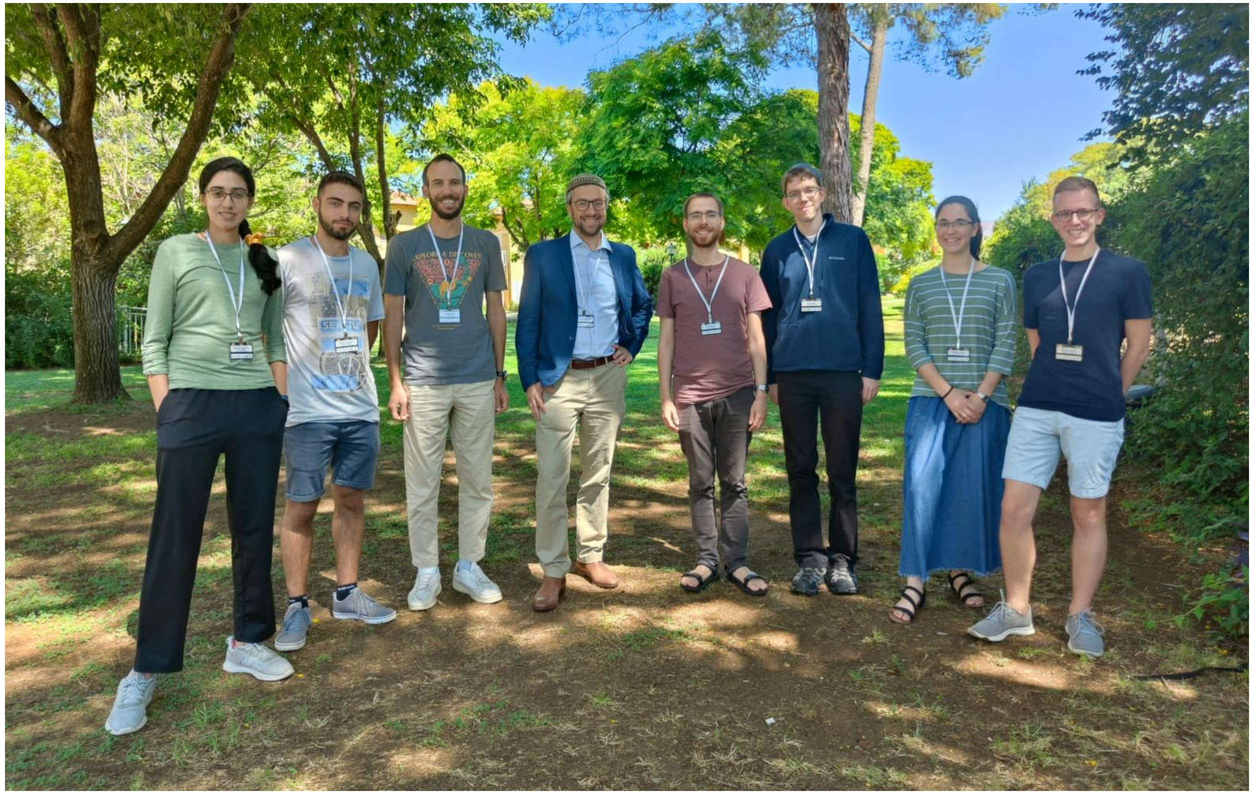
Bar-Ilan University

Bar-Ilan University (established 1955)





Dynamics of complex quantum systems



Michael Stern
(Bar-Ilan)

Itzhack Dana
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Roberta Citro
(Salerno)

Avi Pe’er
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Leon Bello
(→Princeton)

Atanu Rajak
(→Bangalore)

David Dentelski
(→Q. Source)

Inbar Shani
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Marcello Calvanese
Strinati (→Rome)

Yonathan
Saadia (→IDF)

הקרן הלאומית למדע
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Outline

Microwave resonators → coherent Ising machines

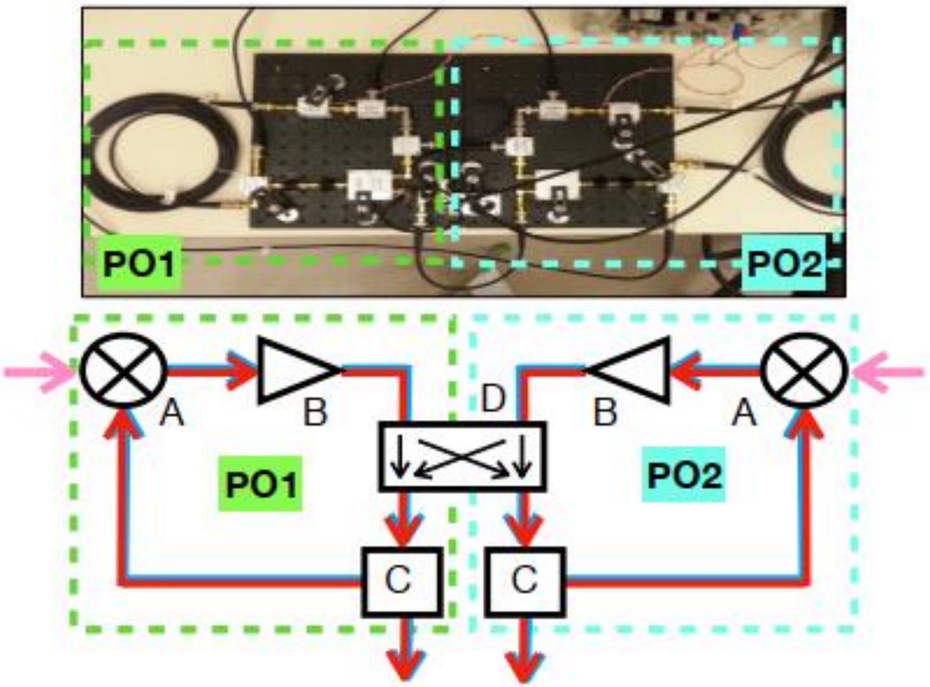
Ultracold atoms

Superconducting circuits

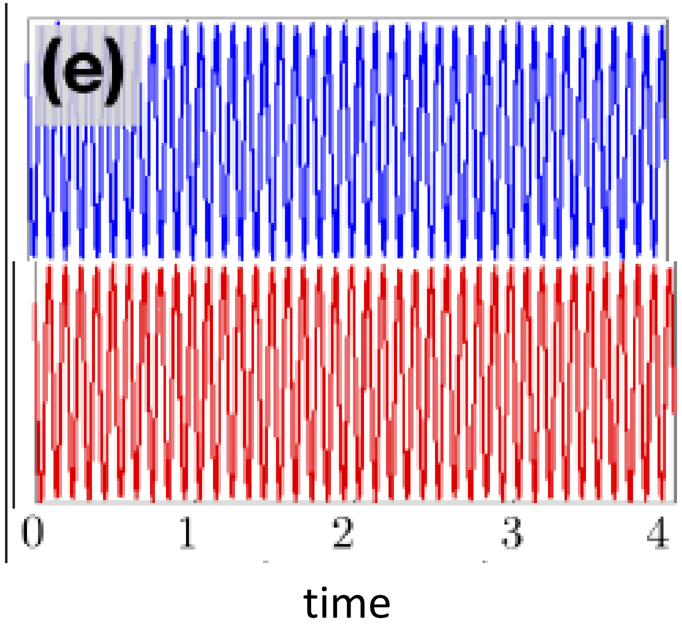
Skipping the introduction to parametric oscillators (PO=bit)

Two coupled parametric oscillators

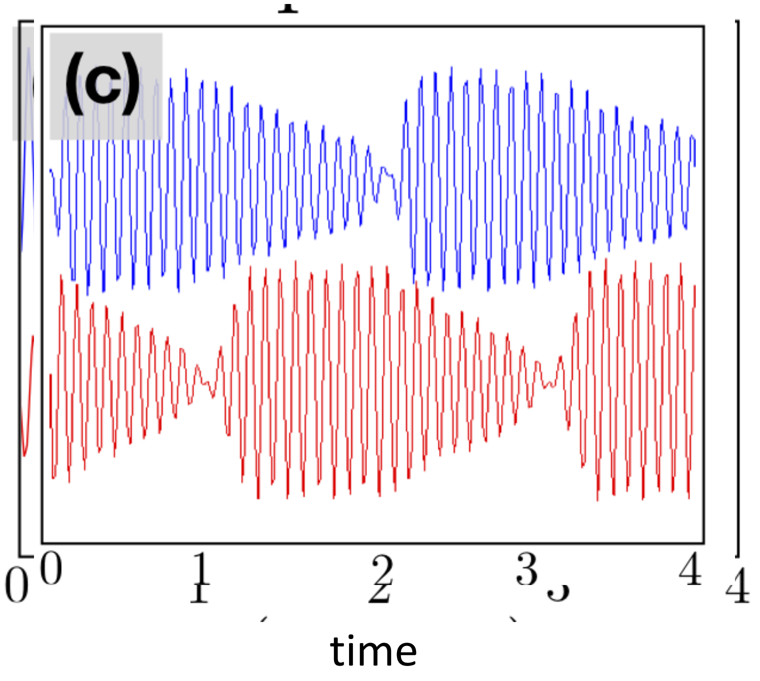
Coaxial cables: microwave oscillators



Naïve expectation



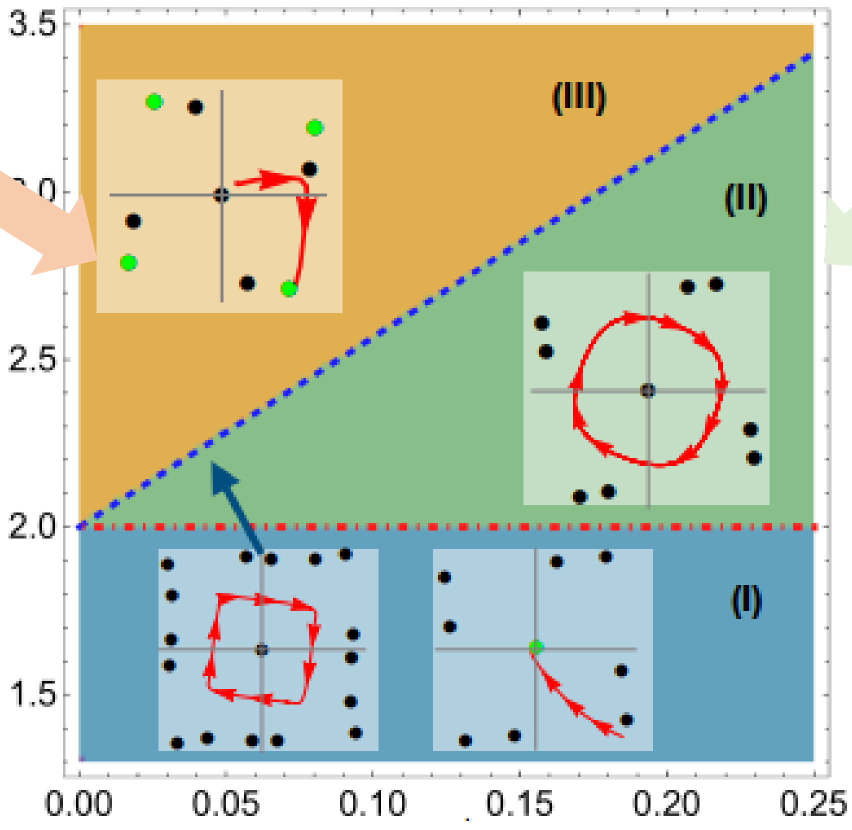
Experiment: beating!



L. Bello, M. Calvanese Strinati, E. G. Dalla Torre, A. Pe'er, PRL&PRA (2020), NJP (2020)

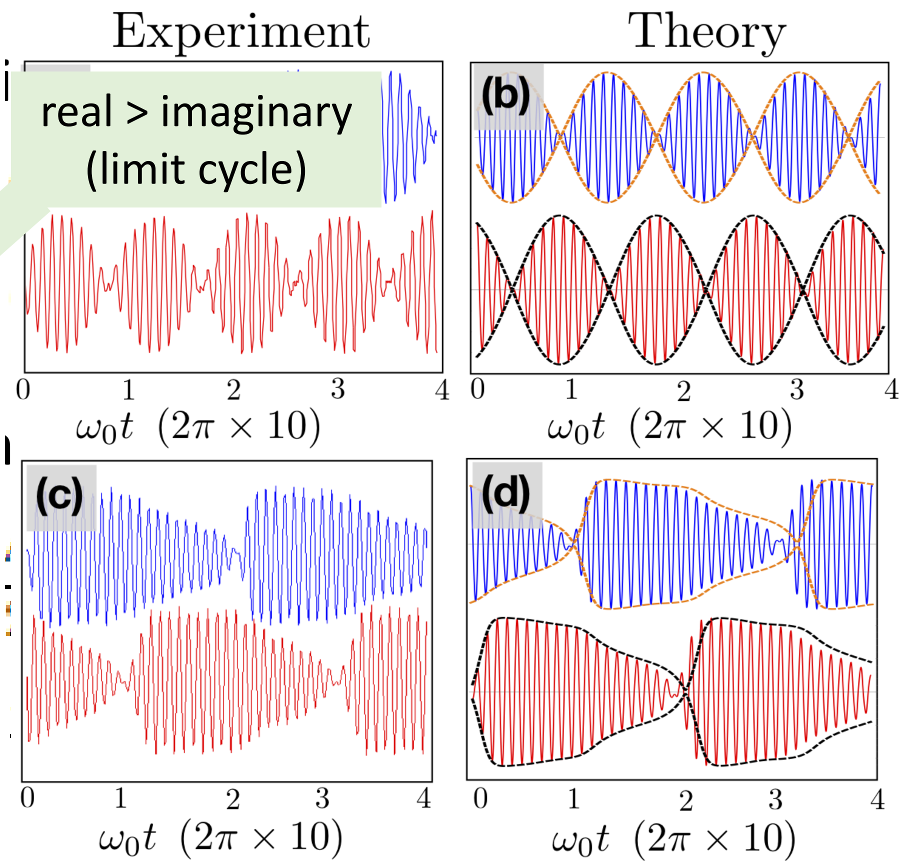
Two coupled parametric oscillators - theory

imaginary > real
(4 attractors)



real coupling /imaginary coupling

real > imaginary
(limit cycle)



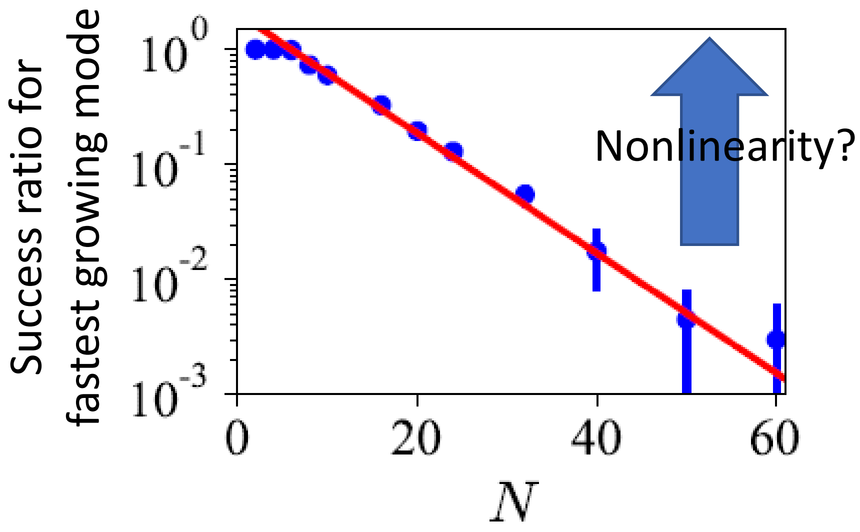
Real coupling (energy conserving) & weak drive→ limit cycles
Imaginary (dissipation), strong drive→ attractors

L. Bello, M. Calvanese Strinati, EGDT, A. Pe'er, PRL&PRA (2020), NJP (2020)

Many coupled PO - Coherent Ising machines (CIM)

Coupling encoded in the dissipative term: $\frac{dE}{dt} = \sum_{i,j} J_{i,j}\sigma_i\sigma_j$

Review: Yamamoto et al, npj quant. info. (2017)

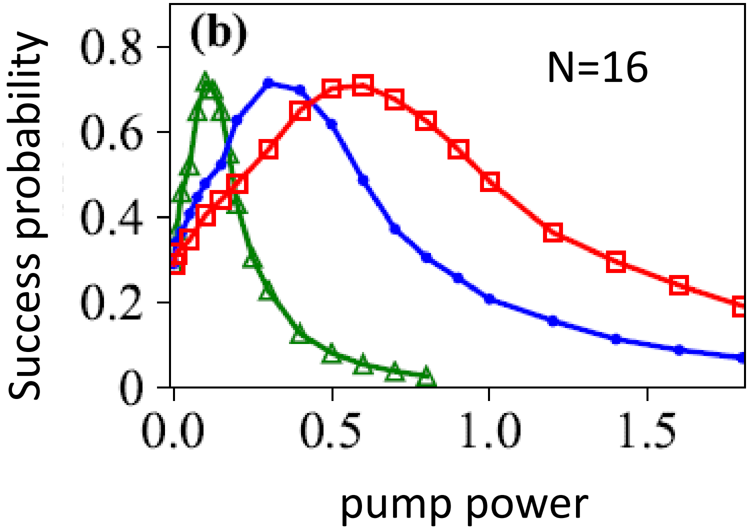


- Linear approximation: mode competition

(BUT finding the maximal eigenvalue of $J_{i,j}$ not NP hard!)

- Nonlinearities can help find the correct solution (heuristics)

M. Calvanese Strinati, L. Bello, EGDT, A. Pe’er, PRL (2021)



Outline

Microwave resonators → coherent Ising machines

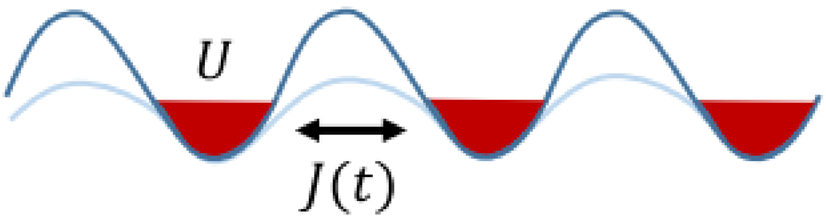
Ultracold atoms → Floquet prethermalization

Superconducting circuits

Periodically driven optical lattices

$$H = \sum_i U\, n_i^2 + J(t)\,(b_i^+ b_j + H.c.)$$

$$J(t) = J_0 + \delta J \cos(\Omega t)$$



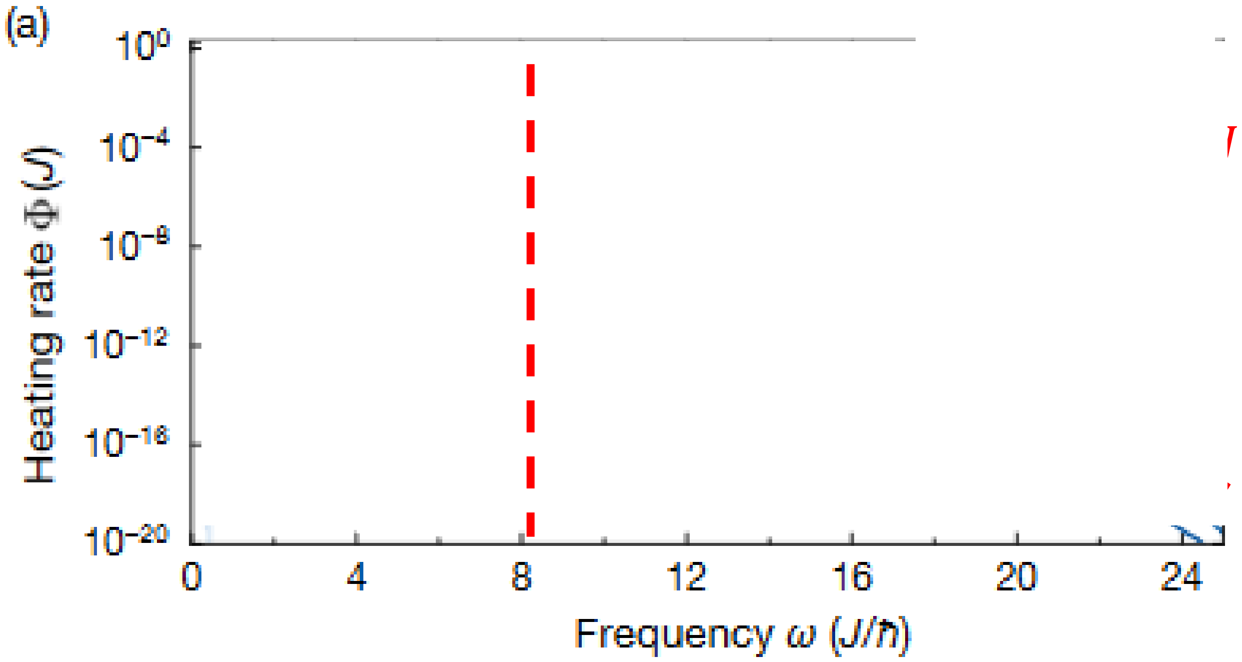
Number and phase

$$H(t) = \sum_i \frac{U}{2} n_i^2 - J(t)\cos(\phi_i - \phi_j)$$

Quadratic approximation:

$$H(t) = \sum_q \quad \frac{U}{2} n_q^2 + 2J(t)\sin(q)\phi_q^2$$

Atanu Rajak, Roberta Citro, EGDT, JPA (2018)

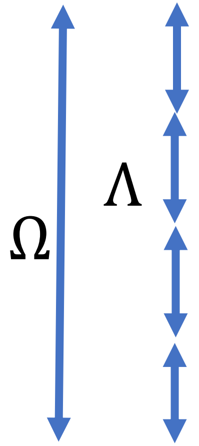


Rubio Abadal et al (I. Bloch group), PRX 2022

Floquet prethermalization

$$\frac{dE}{dt} \sim e^{-\Omega/\Lambda}$$

Rigorous theorems for quantum spin chains



bounded local Hamiltonian

$$\frac{dE}{dt} \sim \epsilon^{\frac{\Omega}{\Lambda}} \sim \exp\left(-\frac{\Omega}{\Lambda}\right)$$

Abanin, De Roeck, Ho, Huveneers (PRL 2015, CMP 2017) -

Kuwahara, Mori, Saito, (Ann.of. Phys 2015, PRL 2015)

Statistical argument for classical systems

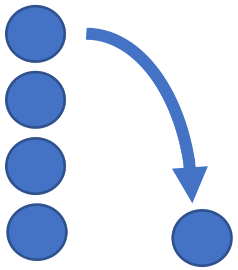
Rajak, Citro, Dalla Torre, JPA (2018)

Rajak, Dana, Dalla Torre, PRB (R) (2019)

Statistical Floquet prethermalization

- Boltzmann distribution → Exponentially low probability of finding a many-body resonance

- Application to the Bose-Hubbard model:



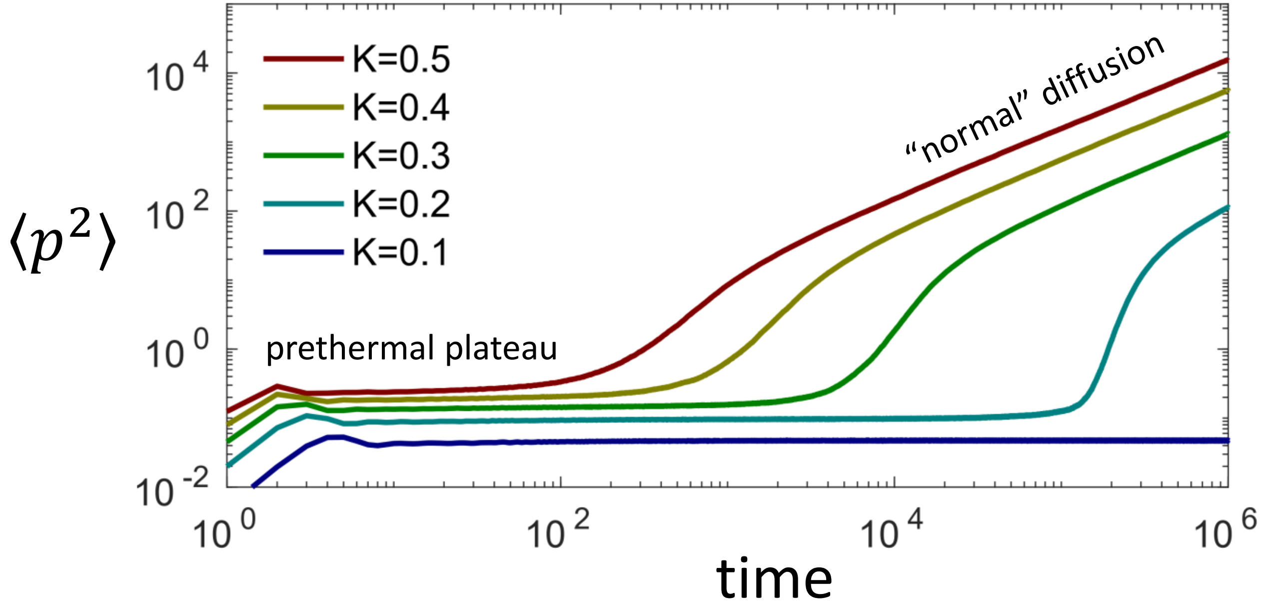
$$E = \frac{U}{2} n(n-1) \Rightarrow \Delta E = U\Delta n$$

EGDT, Dentelski, Scipost 2021

- Transient behavior (coupled rotors)

A. Rajak, R. Citro, EGDT, JPA (2018)
A. Rajak, Dana, Dalla Torre, PRB (R) (2019)

Prethermal timescale $\tau \sim e^{\Omega/\Lambda}$



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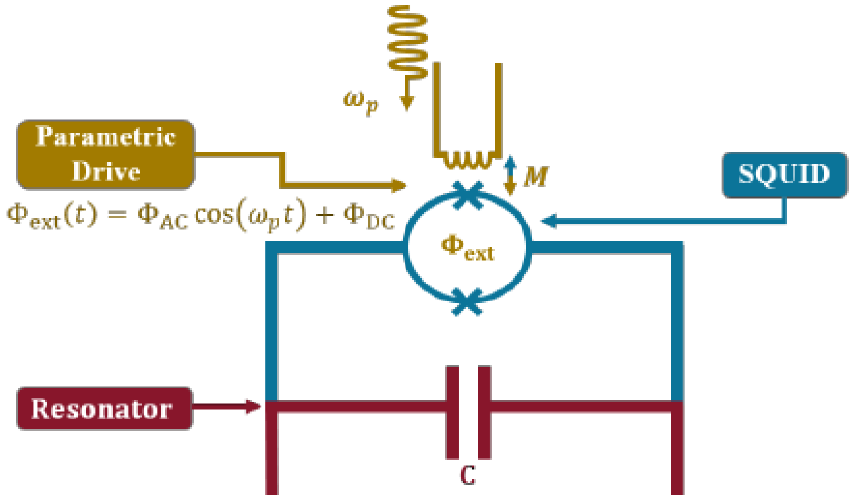
Ultracold atoms → Floquet prethermalization

Superconducting circuits → quantum squeezing

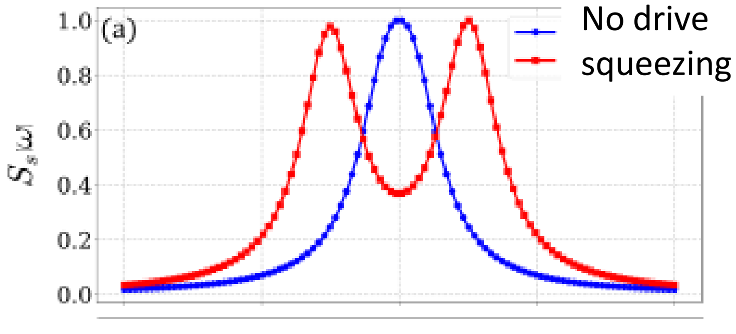
Parametrically pumped resonator (squeezing)



$$\tilde{H}/\hbar = \Omega_r \gamma^\dagger \gamma + \frac{1}{2}\tilde{\omega}_s \sigma_z + \frac{1}{2} g e^{r} (\gamma^\dagger + \gamma)(\sigma_+ + \sigma_-) - \frac{1}{2} g e^{-r} (\gamma^\dagger - \gamma)(\sigma_+ - \sigma_-).$$

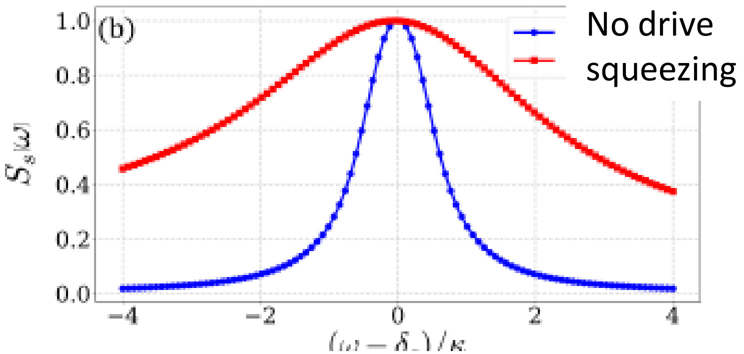
$$\begin{cases} \gamma = a\cosh(r) - a^\dagger \sinh(r) \\ \gamma^\dagger = a^\dagger \cosh(r) - a \sinh(r) \end{cases}$$

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}\left[\tilde{H}, \rho\right] + L\rho L^\dagger - \frac{1}{2}\left(L^\dagger L \rho + \rho L^\dagger L\right)$$



Leroux, Govia, Clerk PRL, 2018

L = squeezed vacuum



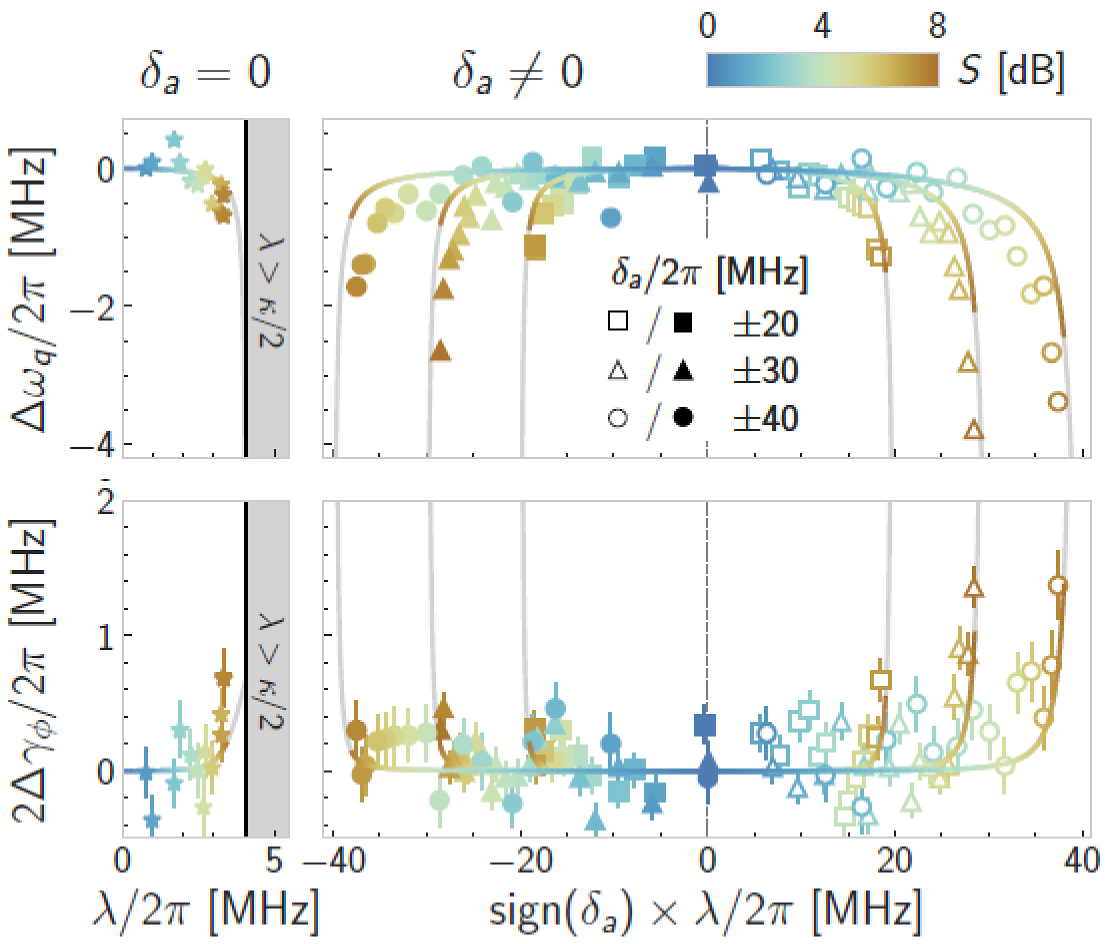
Shani, Dalla Torre, Stern, PRR 2022

L = normal vacuum

Squeezing: two competing effects

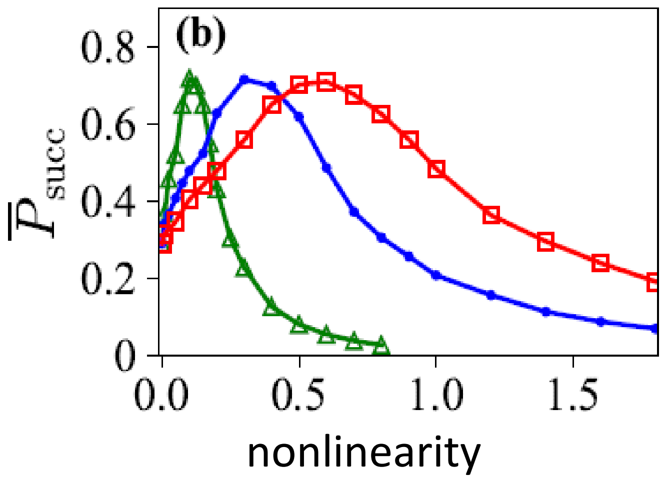
Enhanced dispersive shift:

Enhanced dephasing:



Theory: Inbar Shani, EGDT, Michael Stern, PRR 2021
Experiment: Villiers et al (Kondos&Leghtas group), arxiv

Summary: what did we learn?

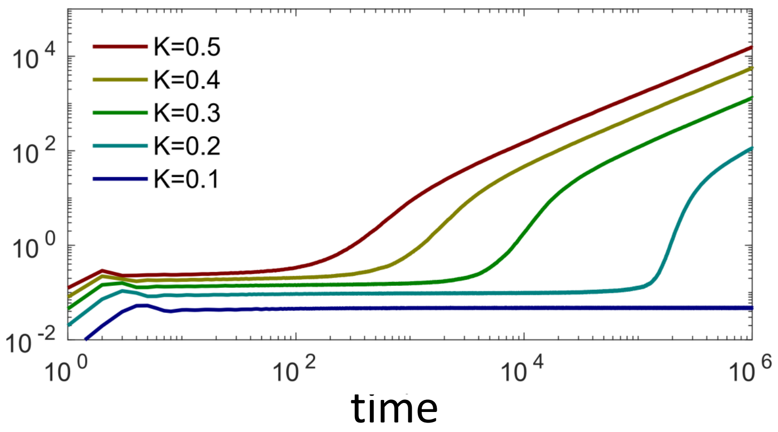


Microwave oscillators → energy-preserving coupling = limit cycles

L. Bello, M. Calvanese Strinati, EGDT, A. Pe'er, PRL&PRA (2020), NJP (2020)

M. Calvanese Strinati, L. Bello, EGDT, A. Pe'er, PRL (2021)

Ultracold atoms → statistical Floquet prethermalization $\frac{dE}{dt} \sim e^{-\frac{\Omega}{\Lambda}}$

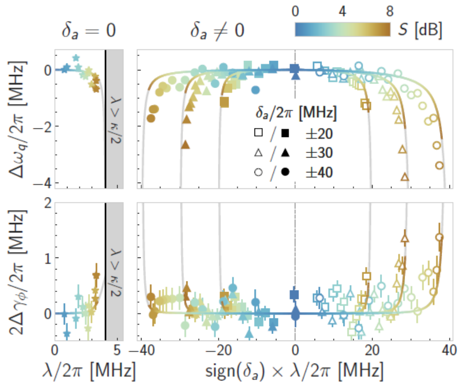


Coupled Oscillators: Citro, Dalla Torre etal, Ann. of Phys., 2015
Rajak, Citro, Dalla Torre, JPA, 2018
Rajak, Dana, Dalla Torre, PRB (R), 2019
Saadia, EGDT, Rajak, PRR, 2019

Bose-Hubbard model: EGDT, Dentelski, Scipost 2021

Review: Wen Wei Ho, Takashi Mori, Dmitry Abanin, EGDT, arxiv 2022

Superconducting circuits → squeezing enhances coupling and dephasing



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