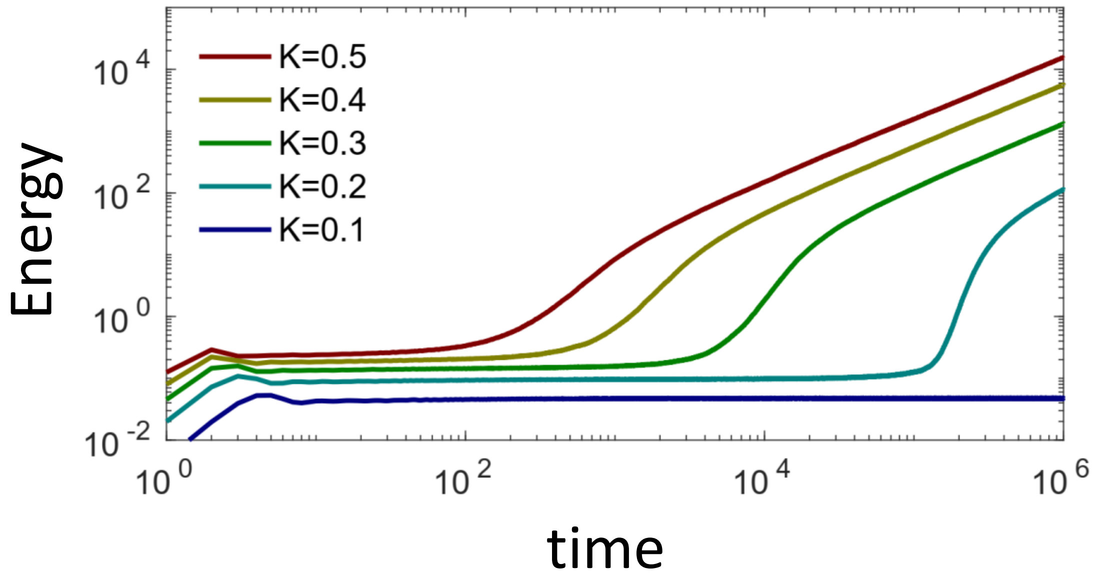
Floquet prethermalization in the (semiclassic) Bose-Hubbard model



Emanuele Dalla Torre

Many-body quantum dynamics group

Marcello Calvanese Strinati

Leon Bello

Daniel Atzes

Inbar Shani

David Dentelski

Yonatan Saadia

Mor Roses

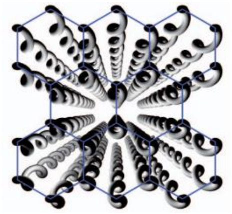
Atanu Rajak (→ Kolkata)

Itzhack Dana
Bar-Ilan University

Roberta Citro
University of Salerno

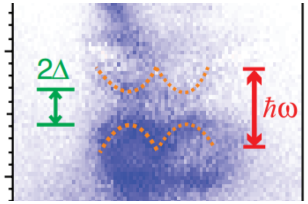
Why periodically-driven systems?

Photonic topological insulators



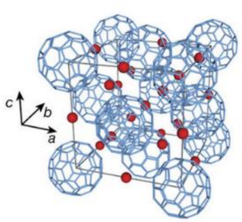
[Segev-Szameit – Nature 2013]

Floquet bands



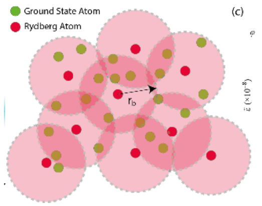
[Gedik – Science 2013]

Light-enhanced superconductivity



[Cavalleri – Nature 2016]

Discrete time crystals



[Lukin – Nature 2017]

Example – Paul trap

Video credit: Harvard Natural Sciences Lecture Demonstrations

The problem : heating

Energy absorption from the drive

↓

Thermalize to infinite temperature

↓

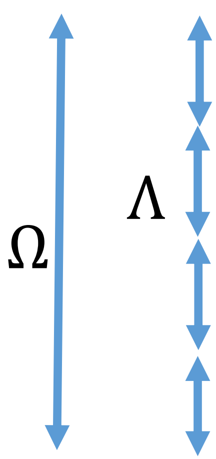
Long time: $Z = e^{\beta H_F} \rightarrow e^{0\, H_F} = 1$

How to prevent heating?

✓ Integrability

✓ Disorder (many-body localization)

✓ **High-frequency drive (Floquet prethermalization)**

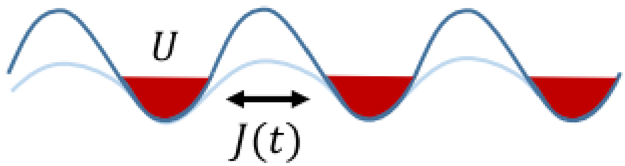


Quantum system with a finite bandwidth

$$\frac{dE}{dt} \sim \epsilon^{\frac{\Omega}{\Lambda}} \sim \exp\left(-\frac{\Omega}{\Lambda}\right)$$

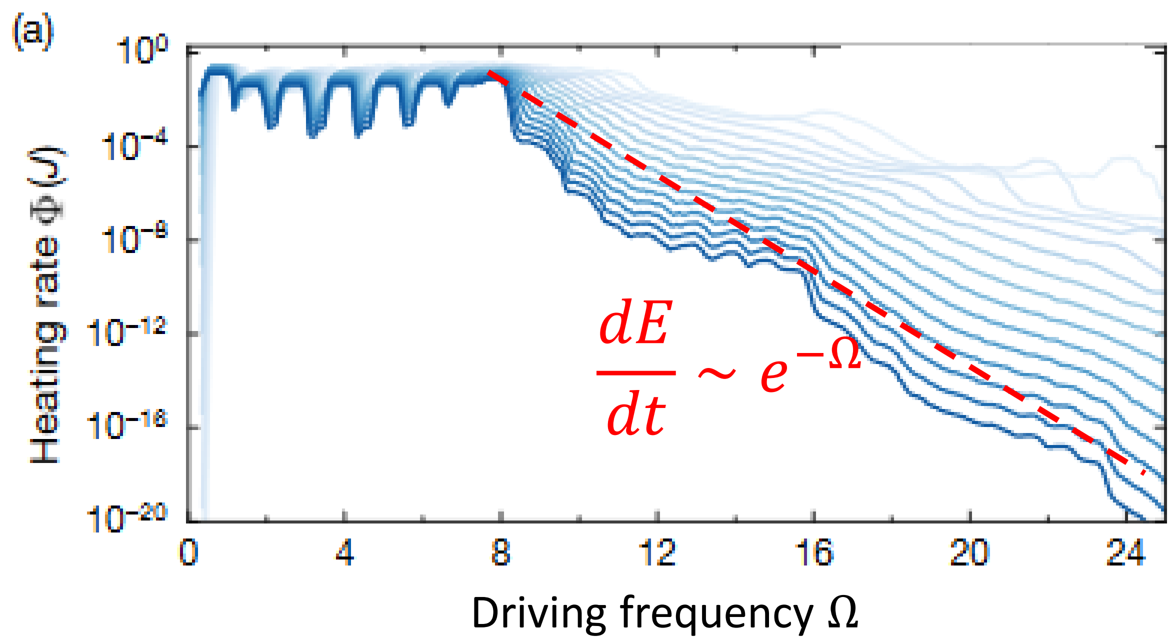
Abanin, De Roeck, Ho, Huveneers (PRL 2015, CMP 2017) - Kuwahara, Mori, Saito, (Ann.of. Phys 2015, PRL 2015)

Prethermalization in the Bose-Hubbard model



$$H = \sum_i U\, n_i^2 + J(t)\,\left(b_i^+ b_j + H.c.\right)$$

See footnote [30] in : Rajak, Dana, Dalla Torre, Phys. Rev. B (R) 100, 100302 (2019)



Rubio Abadal et al (Bloch group) arXiv:2001.08226

How to understand the experiment?

$$H = \sum_i U\, n_i^2 + J(t)\left(b_i^{+} b_j + H.c.\right)$$

~~1. Rigorous theorems for spin systems~~ **Infinite bandwidth**

~~Abanin, De Roeck, Ho, Huveneers (PRL 2015, CMP 2017) - Kuwahara, Mori, Saito, (Ann.of. Phys 2015, PRL 2015)~~

2. Semiclassical approximation

$$[n_i, \phi_i] = i$$

$$H(t) = \sum_i \frac{U}{2} n_i^2 - J(t)\cos(\phi_i - \phi_j)$$

Parametric instability

Citro, Dalla Torre etal, Ann. of Phys. (2015)

Many-body chaos theory

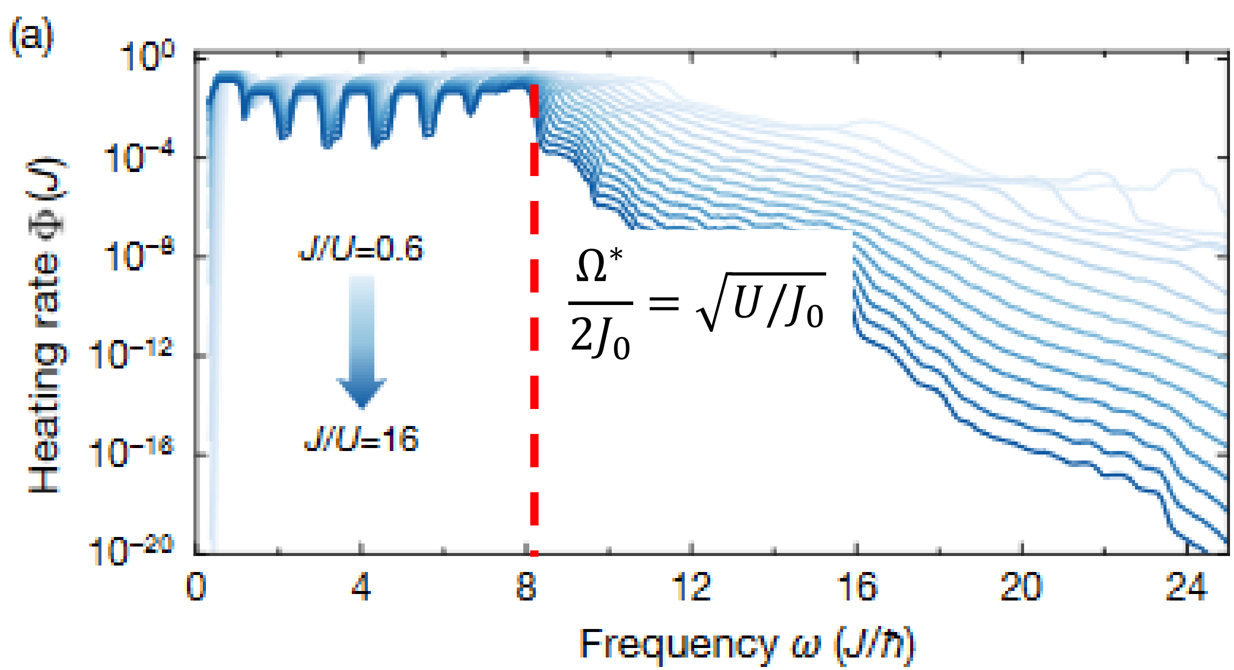
Rajak, Citro, Dalla Torre, JPA (2018)
Rajak, Dana, Dalla Torre, PRB (R) (2019)

Semiclassical approximation : parametric instability

Quadratic approximation:

$$J(t) = J_0 + \delta J \cos(\Omega t)$$

$$H(t) \approx \sum_q \frac{U}{2} n_q^2 + \frac{1}{2} J(t) \sin(q) \phi_q^2$$



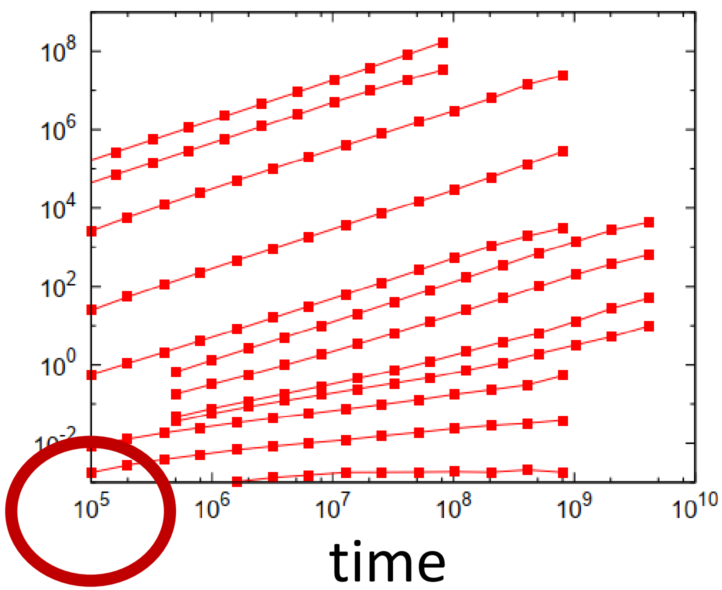
Citro etal (Annals of Physics 2015)

Semiclassical approximation : high frequency

Naively: Infinite bandwidth → No exponential suppression

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right].$$

$\langle p^2 \rangle$

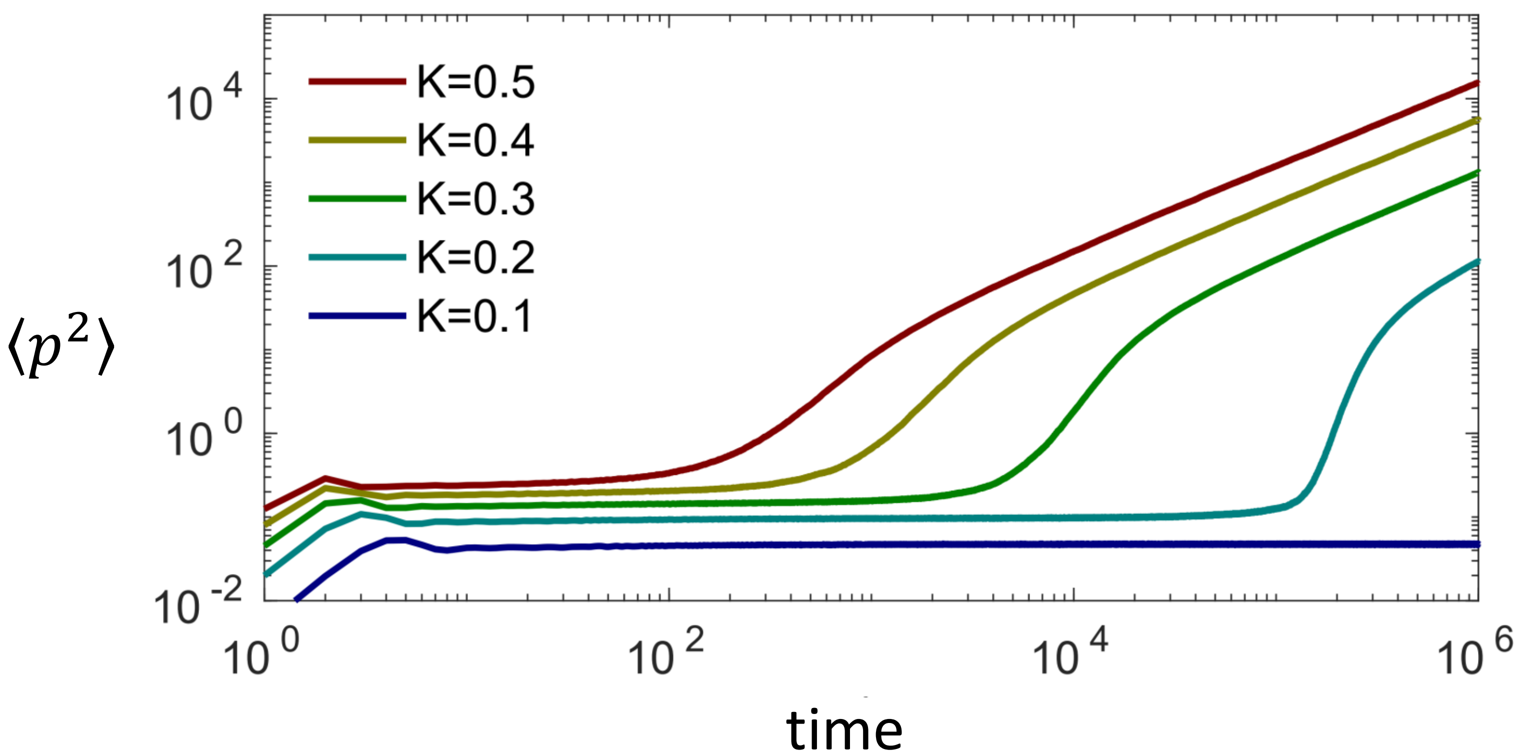


$D \sim \tau^{\alpha} = 1/\Omega^{\alpha}$

Kaneko & Konishi (1989) - Chirikov & Vecheslavov (1997)
Mulanski, Ahnert, Pikovski, Shepelyansky (2011)

What happens at “short” times?

$K = \kappa\tau$



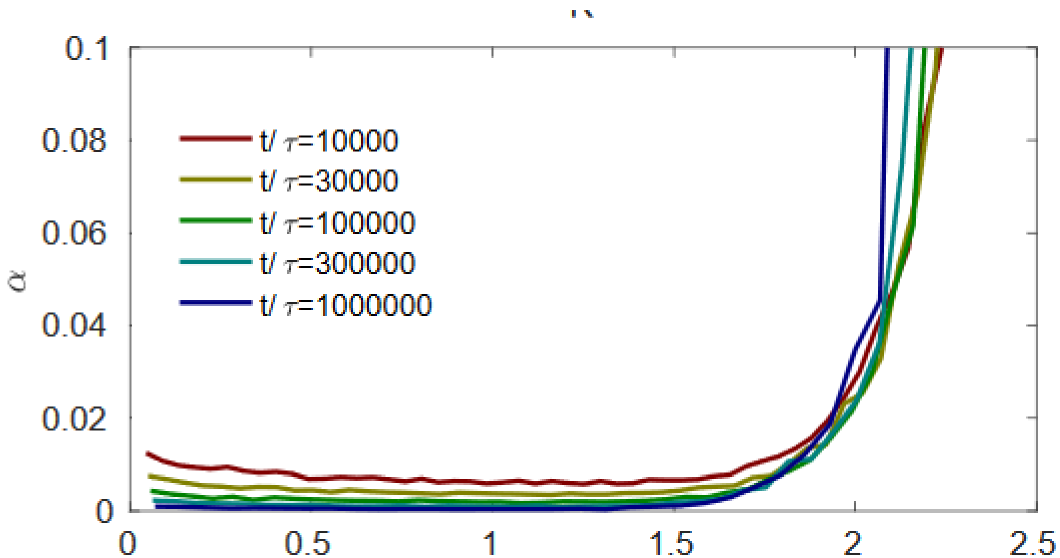
First evidence of Floquet prethermalization in classical systems

A. Rajak, R. Citro, E. G. Dalla Torre, JPA (2018)

See also: Howell, Weinberg, Sels, Polkovnikov, Bukov, PRL (2019)

Lifetime of the prethermal state

$$\langle p(T)^2 \rangle = A\, t^{\alpha}$$



$$\kappa\tau \log\left(\frac{t}{\tau}\right)$$

Lifetime: $\kappa\tau \log\left(\frac{t^*}{\tau}\right) = 2 \;\Rightarrow\; \frac{t^*}{\tau} = \exp\left(\frac{2}{\kappa\tau}\right) = e^{\frac{\Omega}{\kappa}}$

A new mechanism for prethermalization

1. Prethermal state $\boldsymbol{Z = \exp\left(-\frac{H_F}{T^*}\right)}$

$$H_F = \sum_i \frac{p_i^2}{2} + \kappa\Omega\cos(\phi_i - \phi_{i+1})$$

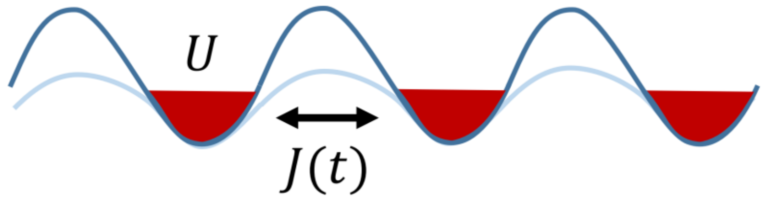
2. Temperature = energy of the initial state : $\boldsymbol{T^* = \kappa\Omega}$

3. Heating = resonance : $\boldsymbol{p_i - p_{i+1} = \mathrm{m}\,\Omega}$

$$Z \sim \exp\left(-\frac{\Omega^2}{2\mathrm{T}^*}\right)$$

$$\frac{t^*}{\tau} \sim \exp\left(\frac{\mathrm{p}^2}{2\mathrm{T}^*}\right) \sim \exp\left(\frac{\Omega}{2\kappa}\right)$$

Main result:



Exponentially long-lived prethermal state in many-body **classical** rotors

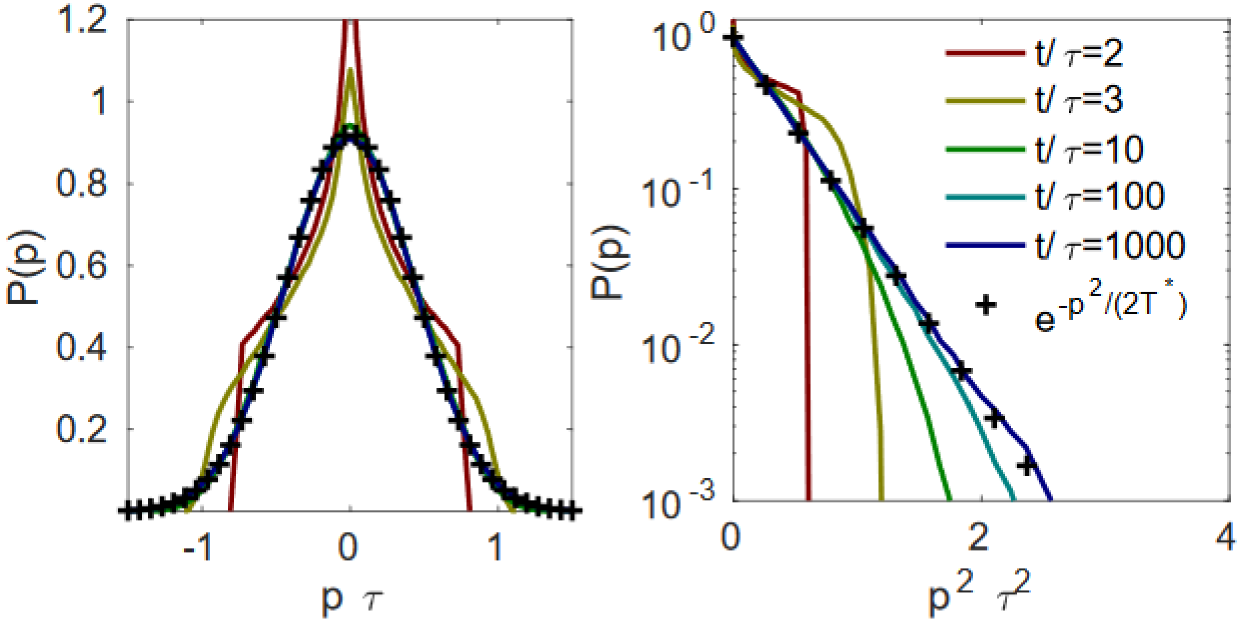
NEW mechanism: low-energy **initial** state

$$\frac{t^*}{\tau} = e^{-\frac{\Omega^2}{2T^*}}$$

Citro, Dalla Torre etal, Ann. of Phys. (2015)
Rajak, Citro, Dalla Torre, JPA (2018)
Rajak, Dana, Dalla Torre, PRB (R) (2019)

Extra slides

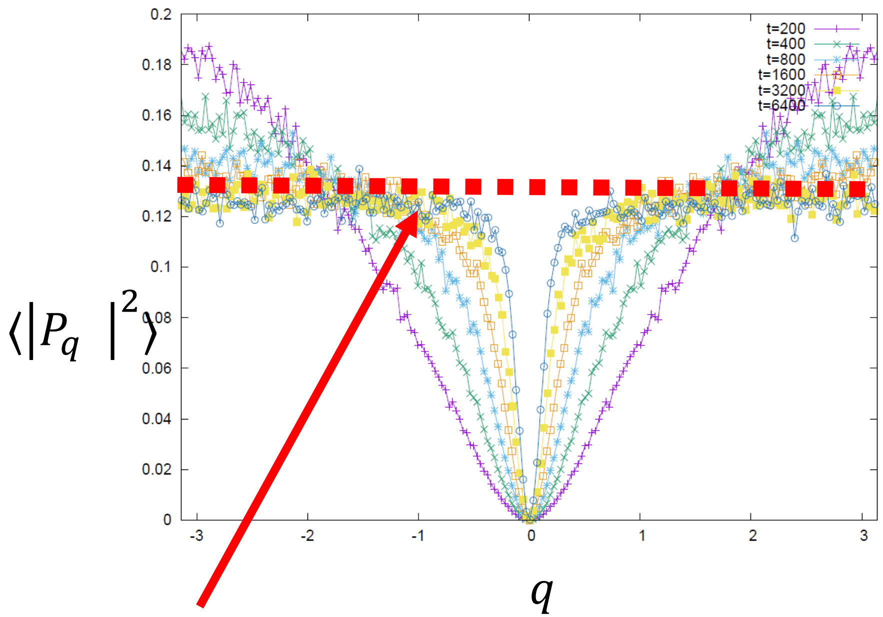
Prethermal state



Coupled kicked rotors – momentum distrib.

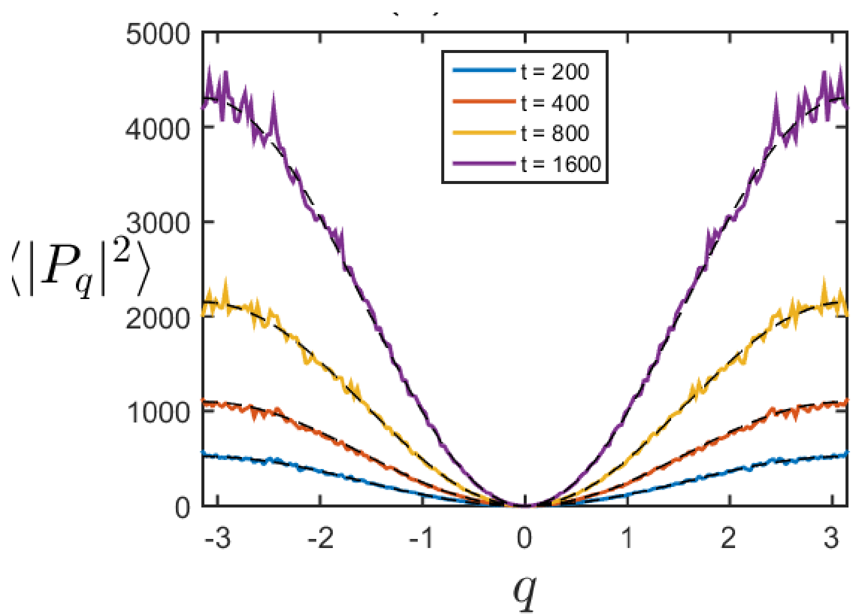
$$P_q = \sum_n e^{iqn} p_n$$

Localized

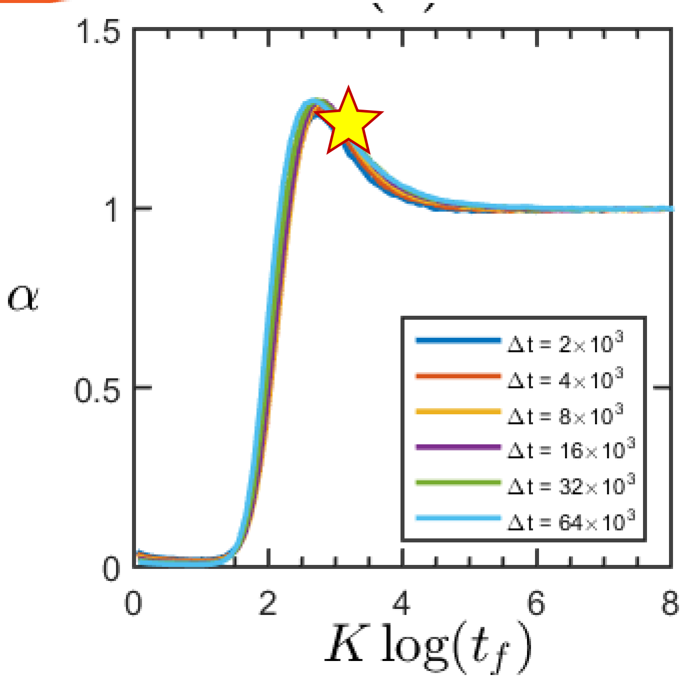


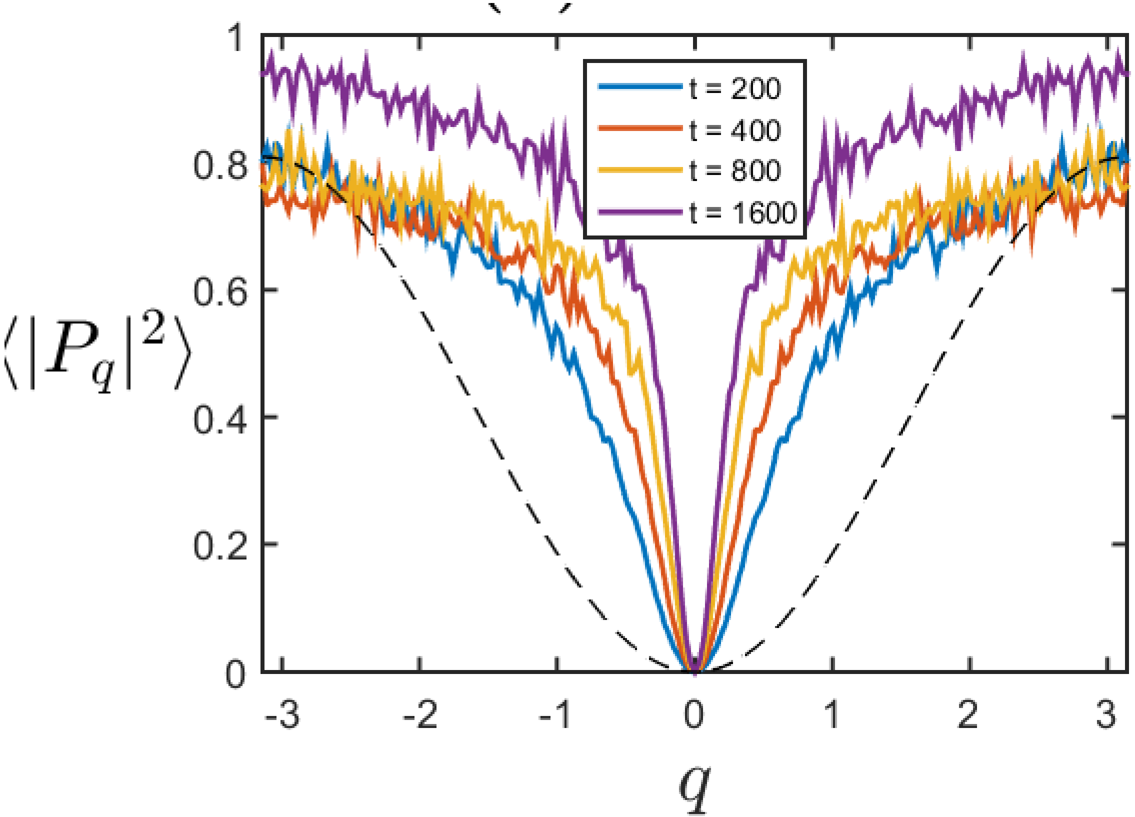
Prethermalization

Diffusive



Super-diffusion





Coupled kicked rotors – quadratic expansion

$$H = \frac{1}{2}\sum_q \left[|P_q|^2 + K(q)|\phi_q|^2 \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right],$$

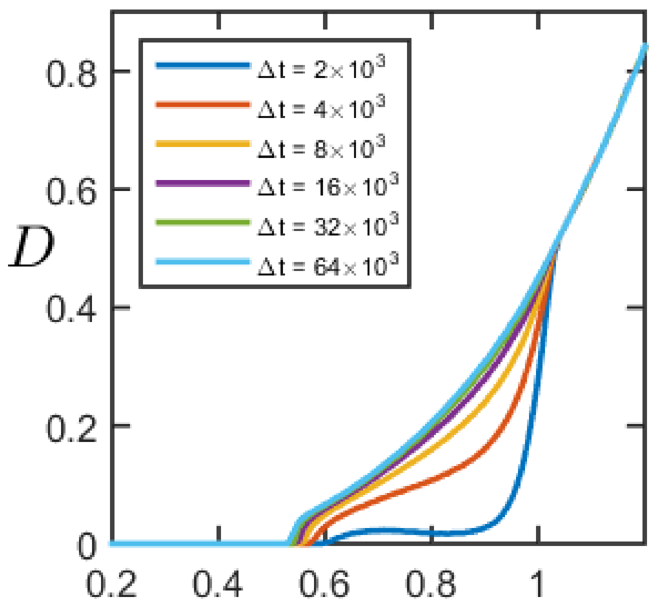
$$K(q) = \frac{4\kappa}{\Omega} \sin^2(\frac{q}{2}).$$

Kicked rotor
("Chirikov standard map")

→ Transition at $\kappa/\Omega = 1$

Coupled kicked rotors – quadratic instability

Initial state: $\phi_j \approx 0$



κ/Ω

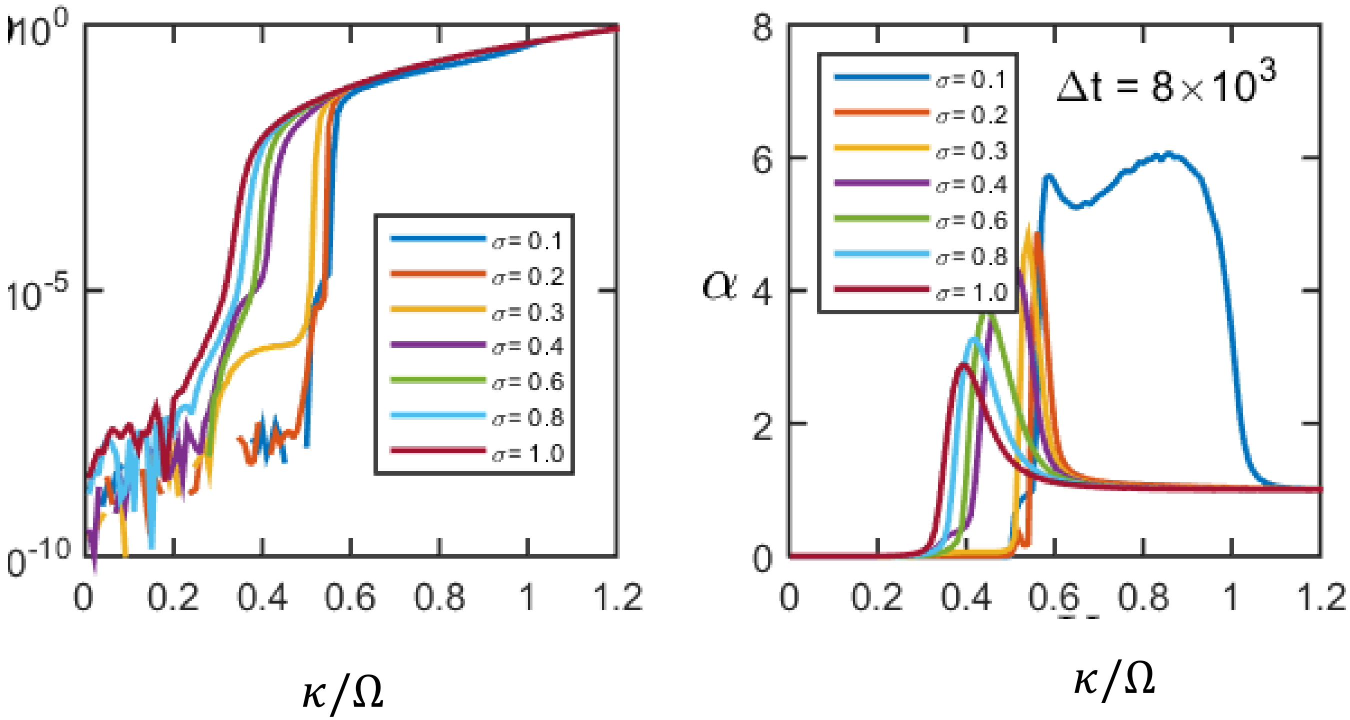
$$H^4 = -\frac{\kappa}{24}\sum_{j=1}^{N}(\phi_j - \phi_{j+1})^4 \sum_{m=1}^{\infty} \cos\left(\frac{2\pi m}{\tau}t\right),$$

$$2\pi m \leq 4\cos^{-1}(1 \; -2\kappa/\Omega)$$

$$m = 4 \;\; \Rightarrow \;\; \frac{\kappa}{\Omega} > 0.5$$

Coupled kicked rotors – from quadratic to marginal

Initial state: $\langle \phi_j^2 \rangle = \sigma$



Finite size effect