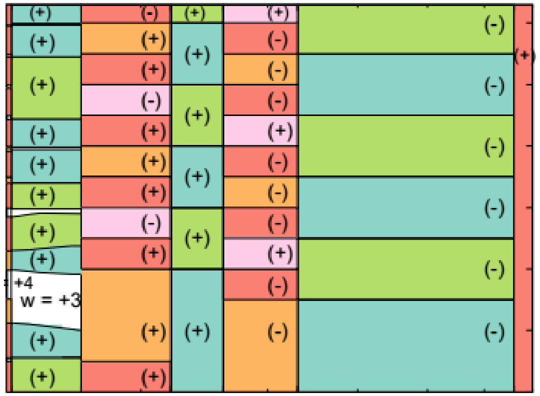
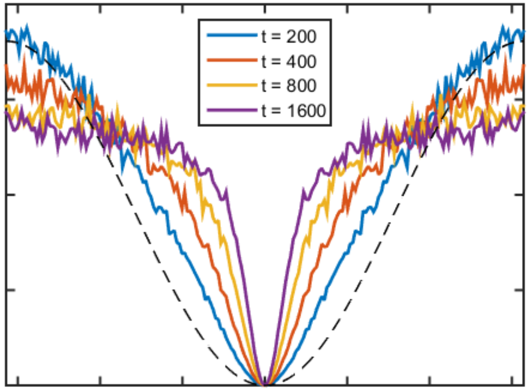
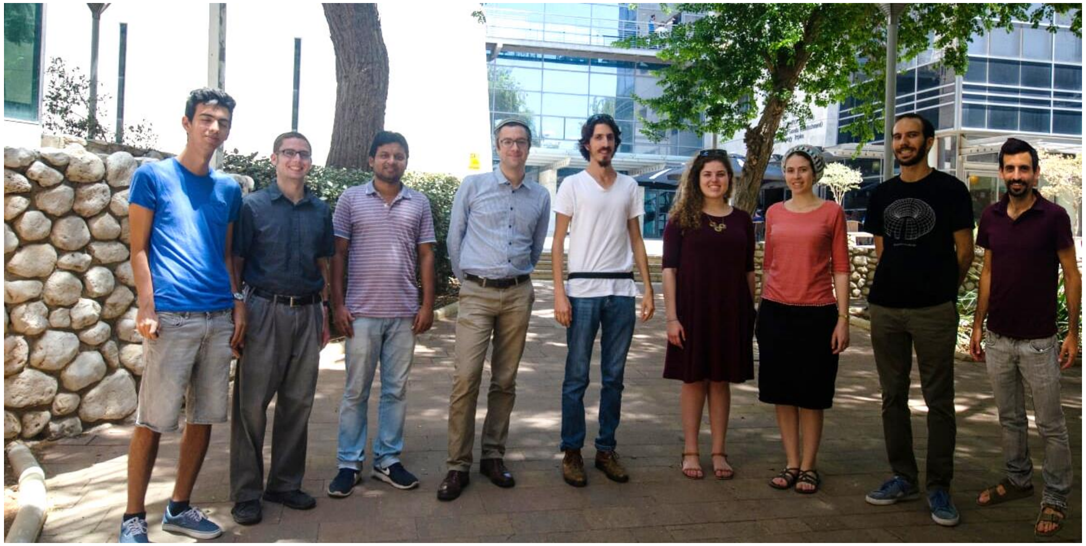
Emanuele Dalla Torre

From Floquet Engineering to Pre-thermalization





Many-body quantum dynamics group



Angelo Russomanno
(→ ICTP, Trieste)

Atanu Rajak
(→ Kolkata)

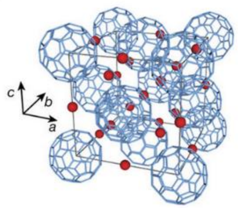
Itzhack Dana
Bar-Ilan University

Roberta Citro
University of Salerno

Why Floquet Engineering ?

Superconductors

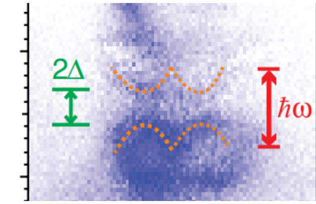
→ Light-enhanced superconductivity



[Cavalleri group – Nature 2016]

Topological insulators

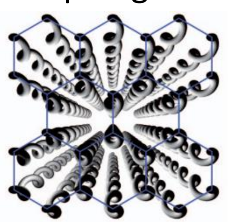
→ Floquet band structure



[Gedik group – Science 2013]

Wave guides

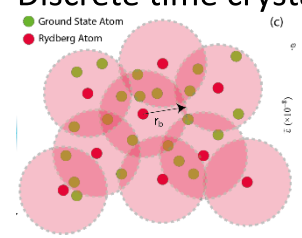
→ Photonic topological insulators



[Segev-Szameit group – Science 2018]

Rydberg atoms

→ Discrete time crystals



[Lukin group – Nature 2017]

Floquet Engineering - idea

Periodically driven Hamiltonian $H(t) = H(t+T)$

⇕

Stroboscopic time evolution $|\psi(nT)\rangle = \left(U(T)\right)^n|\psi_0\rangle$

⇕

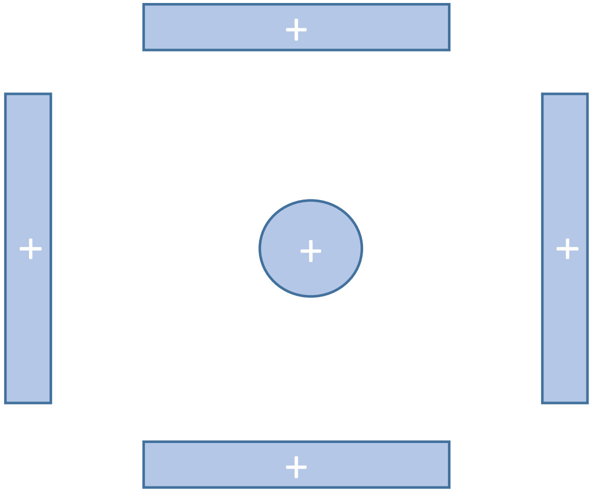
Effective Floquet Hamiltonian $\boxed{U(T) = e^{-i\,H_F\,T}}$

$$|\psi(nT)\rangle = e^{-i\,H_F\,n\,T}|\psi_0\rangle$$

H_F is coooool!

Eckardt, RMP 2017

Example 1 – Paul trap



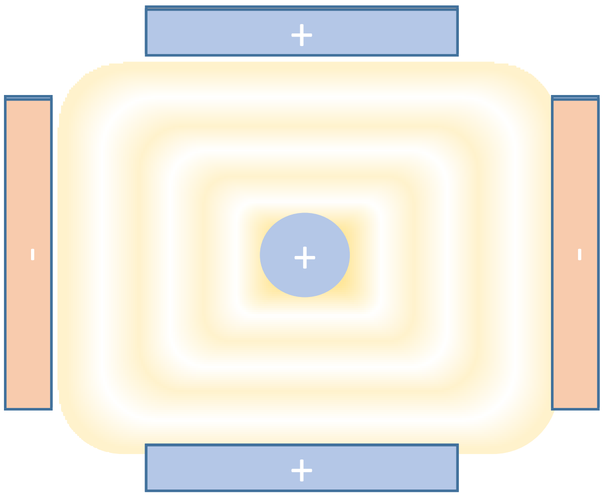
Trapping : $\begin{cases} \vec{\nabla}\phi = 0 \\ \nabla^2\phi > 0 \end{cases}$

But…

$$\nabla^2\phi = -\vec{\nabla}\cdot\vec{E} = -4\pi\rho = 0$$

→ No static trapping in vacuum

Example 1 – Paul trap



$$H(t) = \frac{p^2}{2m} + V(x^2 - y^2)\cos(\Omega t)$$

$$H_{av} = \frac{p^2}{2m}$$

Magnus expansion

$$H_{\mathrm{F}} = H_{av} + \frac{1}{\Omega}\int dt \int dt'[H(t), H(t')] + \ \ldots$$

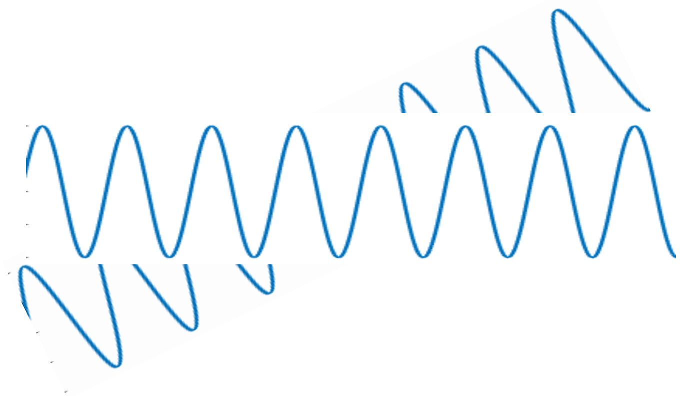
$$H_{\mathrm{F}} \ = \frac{p^2}{2m} + \frac{V^2}{\Omega^2\, m}(x^2 + y^2)$$

Example 1 – Paul trap

Video credit: Harvard Natural Sciences Lecture Demonstrations

Example 2 – lattice shaking

$$H(t) = \sum_i J(c_i^+ c_{i+1} + H.c.) + V\, i\, n_i \cos(\Omega t)$$

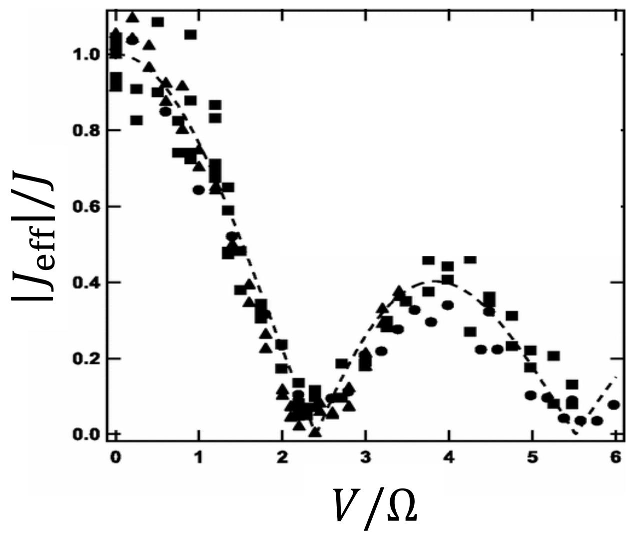


"Gauge transformation" (rotating frame)

$$H(t) = J \sum \left(e^{i\,V\cos(\Omega t)} c_i^+ c_{i+1} + H.c.\right)$$

$$H_{\mathrm{F}} \approx H_{av} = J\, J_0\left(\frac{V}{\Omega}\right) \sum_i (c_i^+ c_{i+1} + H.c.)$$

J_{eff}



Morsch-Arimondo PRL 2007 & PRL 2008

Many-body periodically driven systems

- **Periodically driven Ising model → Time crystal**

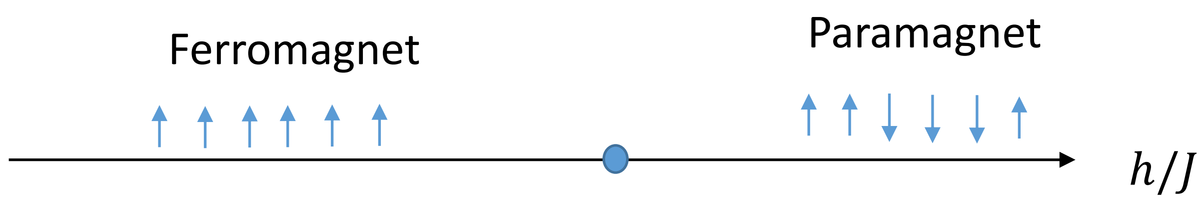
Russomanno, Dalla Torre (EPL 2016)
Russomanno, Friedman, Dalla Torre (PRB 2017)

- **Many body kicked rotor → Prethermalization**

Citro, Dalla Torre, et al, Annals of Physics (2015)
Rajak, Citro, Dalla Torre (J. Phys. A: Math. Theor. 51 465001 (2018))
Rajak, Dana, Dalla Torre (in prep.)

Ising model : static

$$\hat{H}(t) = \sum_{j:} \left(-J\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + h(t)\hat{\sigma}_j^x\right) \qquad h(t) = h_0$$

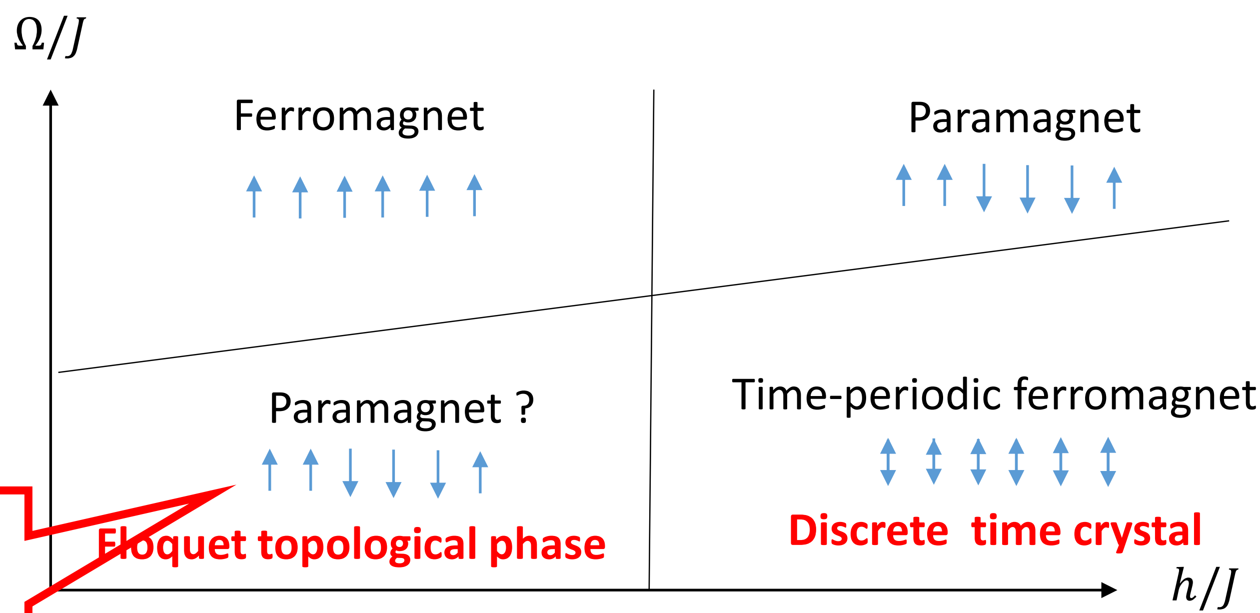


Quantum phase transition

Ising model : periodic drive

$$\hat{H}(t) = \sum_{j} \left(-J\hat{\sigma}^z_j\hat{\sigma}^z_{j+1} + h(t)\hat{\sigma}^x_j\right)$$

$$h(t) = h_0 + \delta h \cos(\Omega t)$$



Non local order

$$\hat{\beta}^y_j = \left(\prod_{k=1}^{j-1} \hat{\sigma}^x_k\right) \hat{\sigma}^z_j \hat{\sigma}^y_{j+1}$$

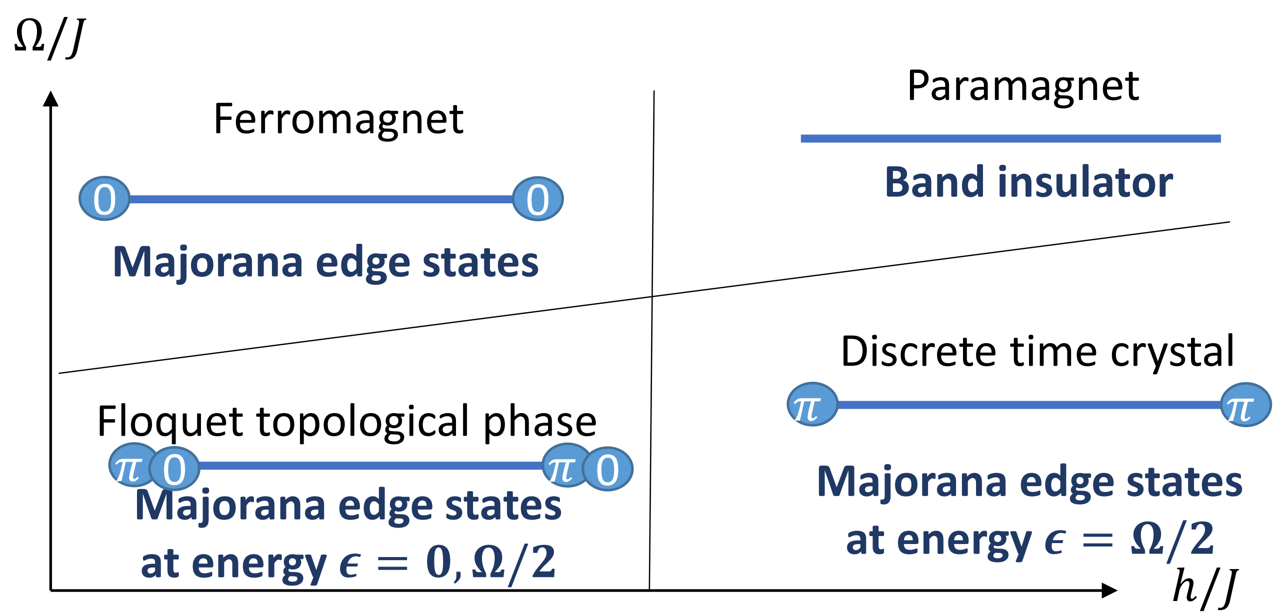
Else, Bauer, Nayak (PRL 2016)

Russomanno, Friedman, Dalla Torre (PRB 2017)

Ising model : periodic drive

Jordan-Wigner transformation

$$\hat{H}(t) = \sum_{j} \left(-J\hat{\sigma}^z_j\hat{\sigma}^z_{j+1} + h(t)\hat{\sigma}^x_j\right) \Rightarrow H = \sum_j J\,(c_i + c_i^\dagger)(c_{i+1} - c_{i+1}^\dagger) + h(t)c_i^\dagger c_i$$



End of the story?

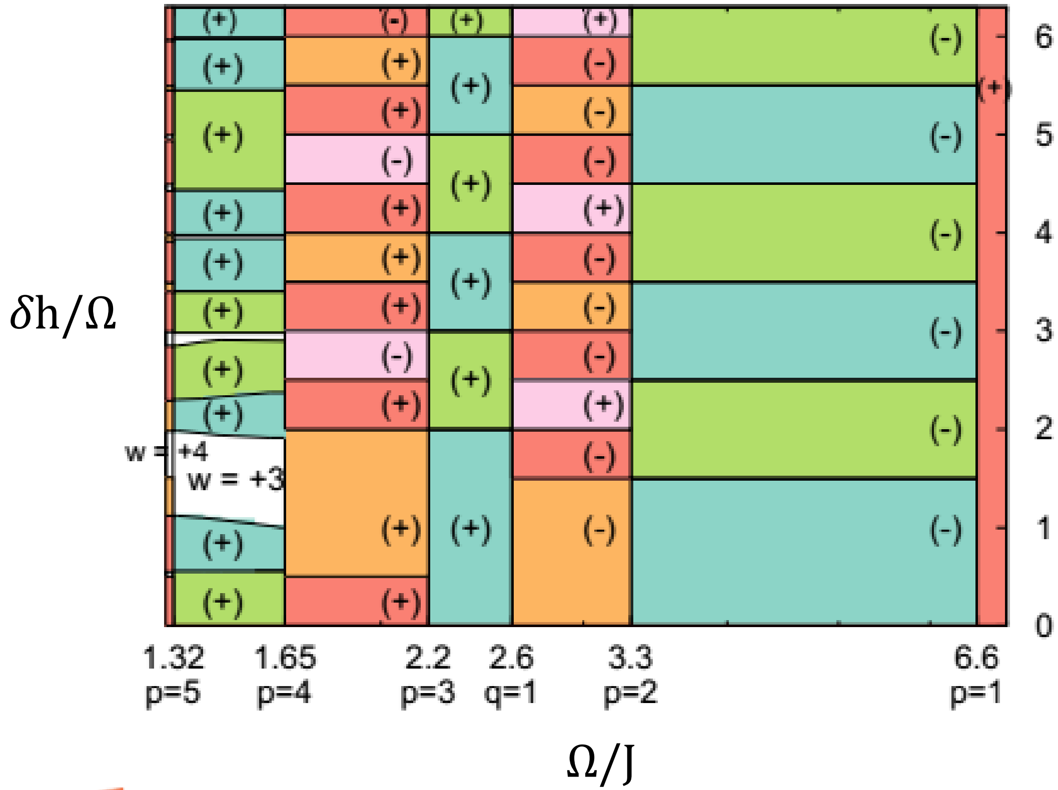
Khemani, Lazarides, Moessner, Sondhi PRL 2015

Else, Bauer, Nayak PRL 2016

Ising model : periodic drive

$$\hat{H}(t) = \sum_{j} \left(-J\hat{\sigma}^z_j\hat{\sigma}^z_{j+1} + h(t)\hat{\sigma}^x_j\right)$$

$$h(t) = 2.3\,J + \delta h\cos(\Omega\, t)$$



- **Paramagnet**
- **Ferromagnet**
- **Time crystals**
- **Topological phases**
- **Floquet topological phase**

Russomanno, Dalla Torre (EPL 2016)
Russomanno, Friedman, Dalla Torre (PRB 2017)

The problem : heating

Break integrability = Add interactions

↓

Thermalize (Eigenstate thermalization hypothesis)

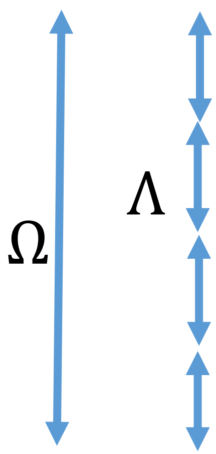
↓

Steady state = infinite temperature

$$Z = e^{\beta H_F} \rightarrow e^{0\, H_F} = 1$$

How to prevent heating?

- ✓ Integrability
- ✓ Disorder (many-body localization)
- ✓ **High-frequency drive (prethermalization)**



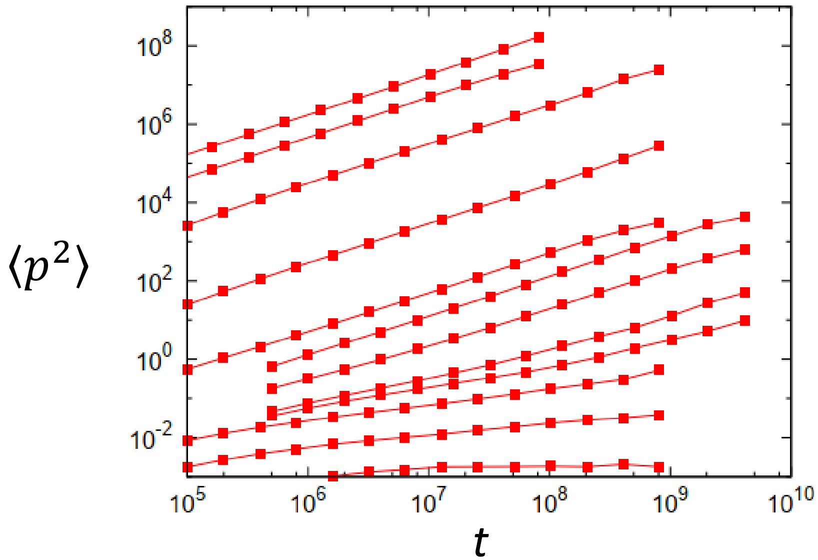
Quantum system with a finite bandwidth

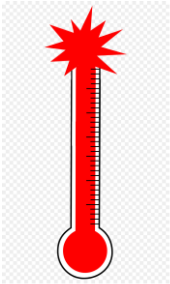
$$\frac{dH}{dt} \sim \epsilon^{\frac{\Omega}{\Lambda}} \sim \exp\left(-\frac{\Omega}{\Lambda}\right)$$

Abanin *etal* (PRL 2015) , (CMP 2017) - Kuwahara, Mori, Saito, (Ann.of. Phys 2015), (PRL 2015)

Classical systems with infinite bandwidth?

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right].$$

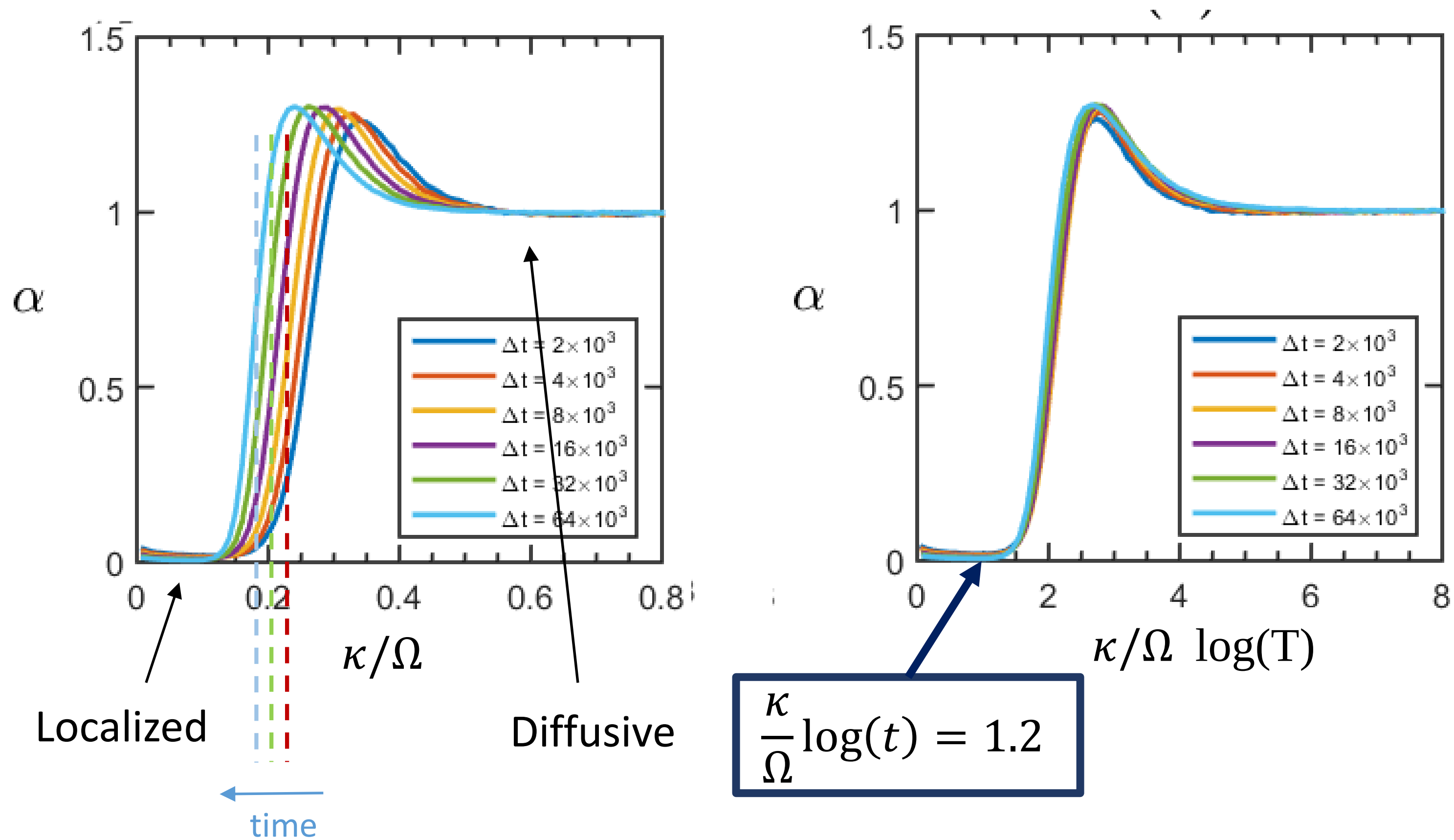




Kaneko & Konishi (1989) - Chirikov & Vecheslavov (1997)- Mulanski, Ahnert, Pikovski, Shepelyansky (2011)

Coupled kicked rotors – our results

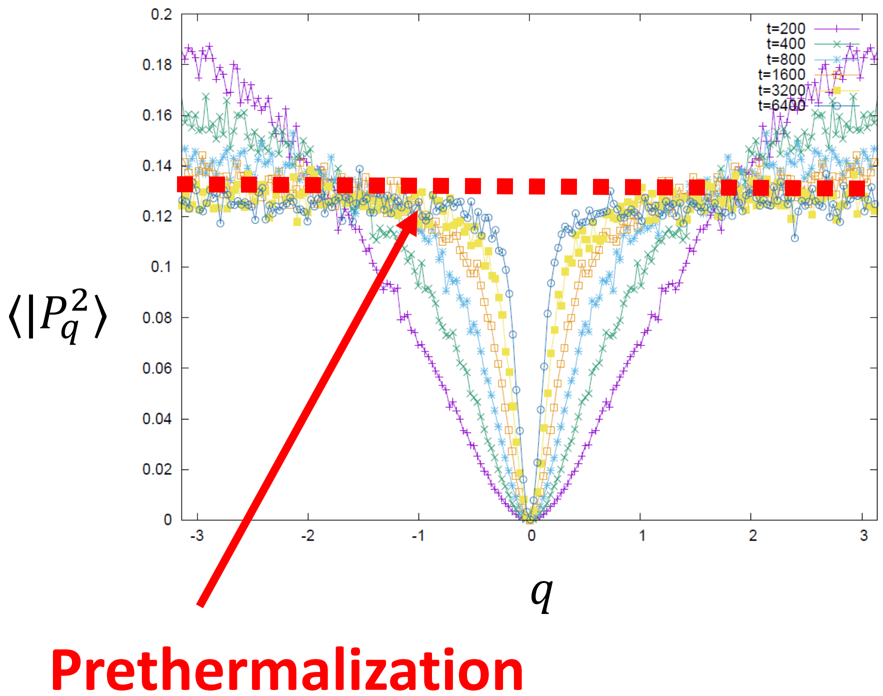
$$\langle p(T)^2 \rangle = A\, t^{\alpha}$$



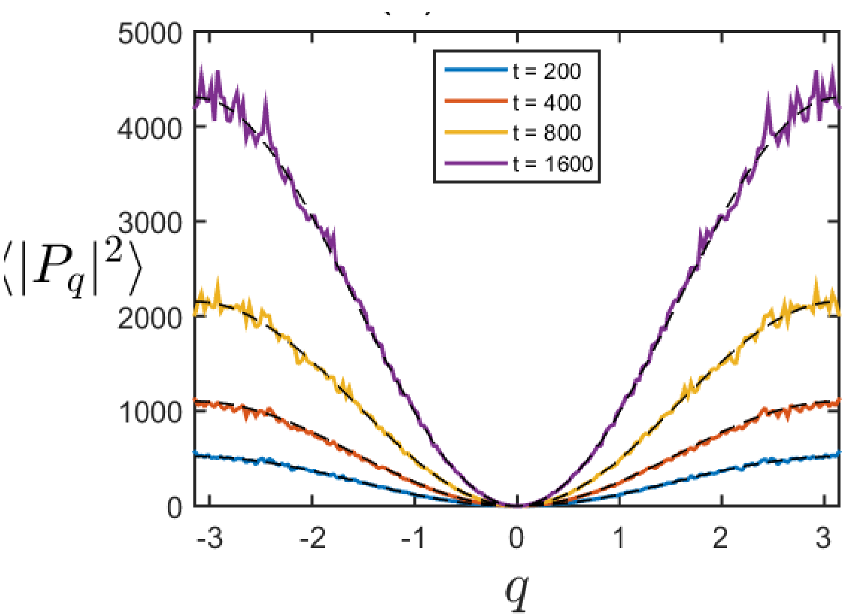
Coupled kicked rotors – momentum distrib.

$$P_q = \sum_n e^{iqn} \phi_n$$

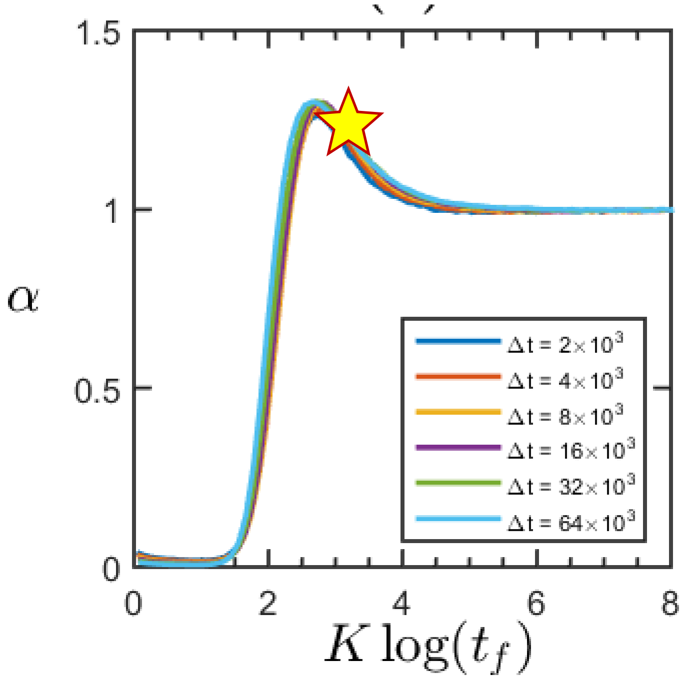
Localized

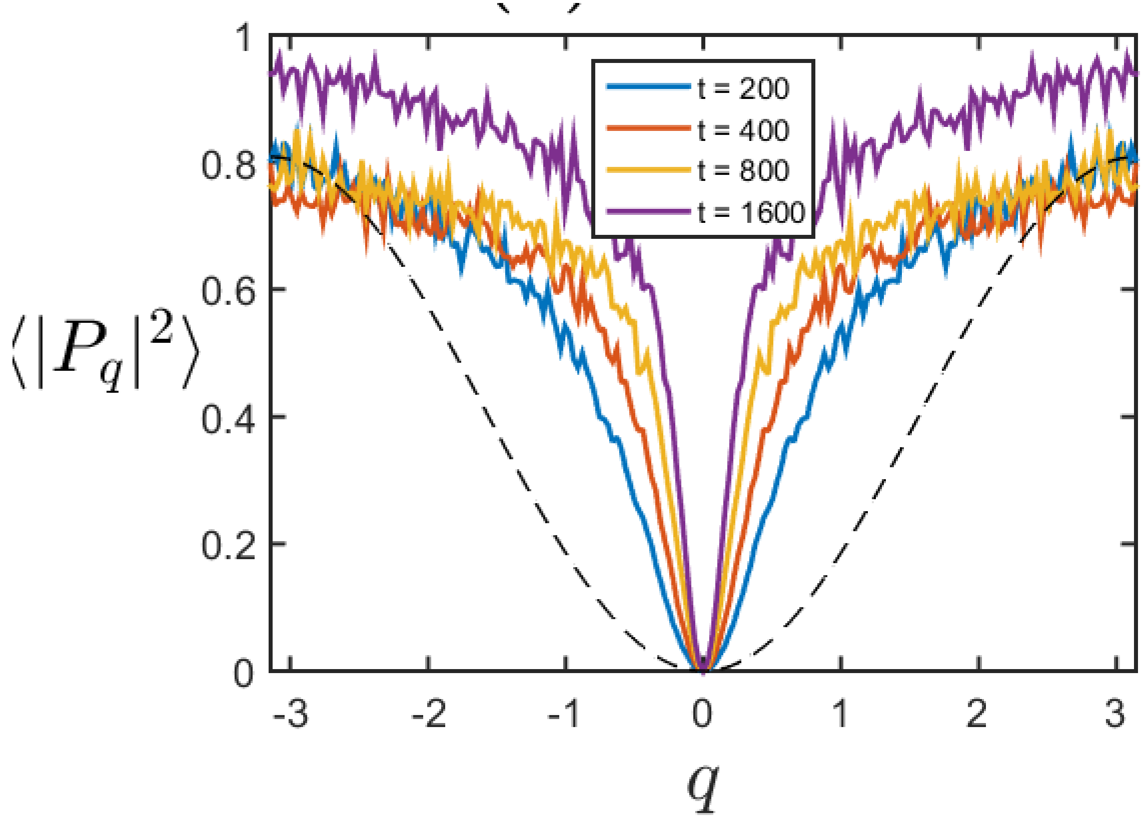


Diffusive



Super-diffusion





Prethermalization in classical systems

$$H(t) = \sum_i \frac{p_i^2}{2} + K\Delta(t)\cos(\phi_i - \phi_{i+1})$$

$$\Delta(t) = \sum_n \delta(t - n\frac{2\pi}{\Omega})$$

Localizations threshold

$$\frac{\kappa}{\Omega}\log(\Omega\, t^*) = 1.2 \quad \Rightarrow \quad \Omega\, t^* = \exp\left(1.2\,\frac{\Omega}{\kappa}\right)$$

$$\Omega \to \infty$$

~~High-frequency suppression of heating (Floquet Hamiltonian)~~

$$K \to 0$$

~~Reduced diffusion close to integrability (Arnold diffusion)~~

Prethermalization in classical systems : 1,2,3

1. Boltzmann distribution : $Z = \exp\left(-\frac{H_F}{T}\right)$

$$H_F = \sum_i \frac{p_i^2}{2} + \kappa\Omega\cos(\phi_i - \phi_{i+1})$$

2. Temperature = energy of the initial state : $T = \kappa\Omega$

3. Heating = resonance : $p_i - p_{i+1} = \mathrm{m}\,\Omega$

$$\Rightarrow \quad \frac{dH}{dt} \sim \exp\left(-\frac{\Omega}{\kappa}\right)$$

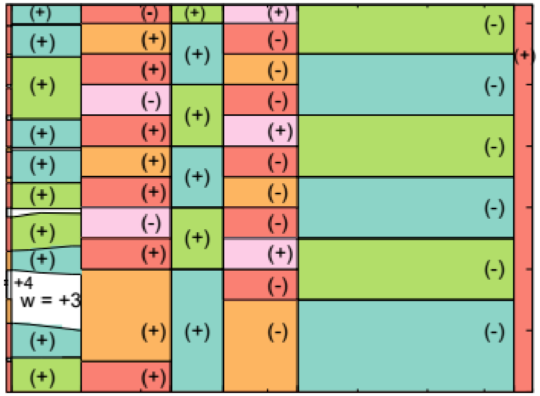
From Floquet Engineering to Pre-thermalization

Periodically driven spin chain

- **Periodically driven *quantum* Ising model**

  → **Time crystal & Floquet topological phases**

  Russomanno, Dalla Torre (EPL 2016)
  Russomanno, Friedman, Dalla Torre (PRB 2017)



- **Many body *classical* kicked rotor**

  → **Localization & Prethermalization**

  Citro, Dalla Torre, et al, Annals of Physics (2015)
  Rajak, Citro, Dalla Torre (J. Phys. A: Math. Theor. 51 465001 (2018))
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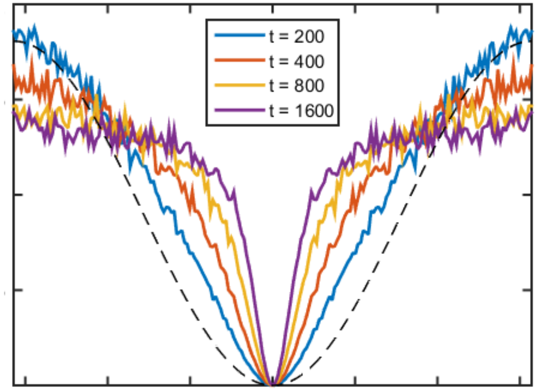


Many-body quantum dynamics group

Classical Prethermalization

Dr. Atanu Rajak:
Dynamic localization of
a many-body kicked rotor

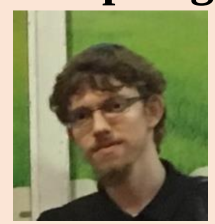
Quantum optics

Mor Roses:
Dicke superradiance
and counterlasing

Nonlinear optics
(with Avi Pe'er)

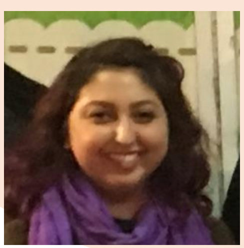
Dr. Marcello Strinati:
Ising simulators with
parametric amplifiers

Topology in one dimension



Daniel Atzitz:
Symmetry resolved
entangled states

Superconducting circuits
(with Michael Stern)



Inbar Shani:
Parametrically amplified
spin-cavity couplings

Strongly correlated materials

David Dentelski:
Short vs. Long-range
superconducting fluctuations

From Floquet Engineering to Pre-thermalization

Periodically driven spin chain

- **Periodically driven *quantum* Ising model**

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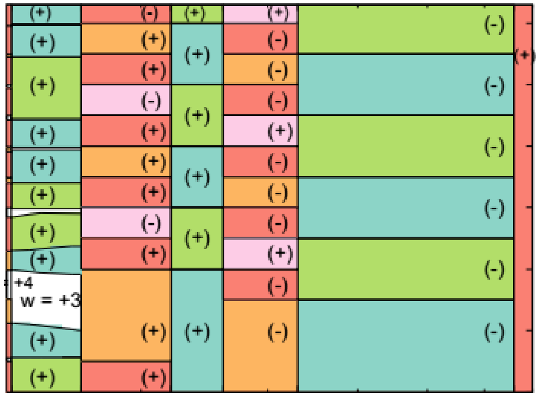

- **Many body *classical* kicked rotor**

  → **Localization & Prethermalization**

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  Rajak, Citro, Dalla Torre (arXiv:1801.01142)
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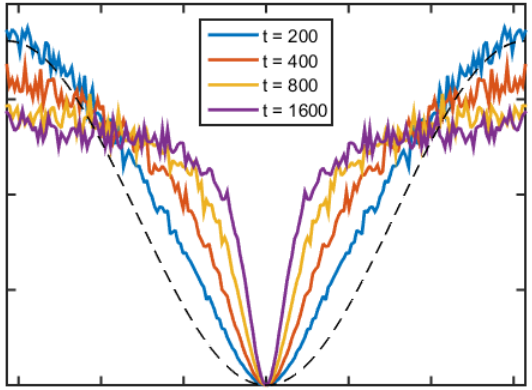


Extra slides

Coupled kicked rotors – quadratic expansion

$$H = \frac{1}{2}\sum_q \left[|P_q|^2 + K(q)|\phi_q|^2 \sum_{n=-\infty}^{+\infty} \delta(t - n\tau)\right],$$

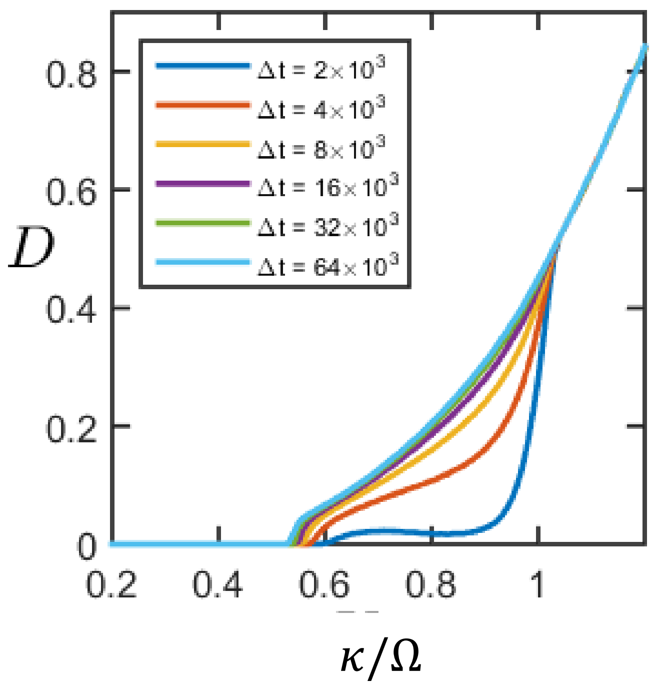
$$K(q) = \frac{4\kappa}{\Omega}\sin^2(\frac{q}{2}).$$

Kicked rotor
("Chirikov standard map")

→ Transition at $\kappa/\Omega = 1$

Coupled kicked rotors – quadratic instability

Initial state: $\phi_j \approx 0$



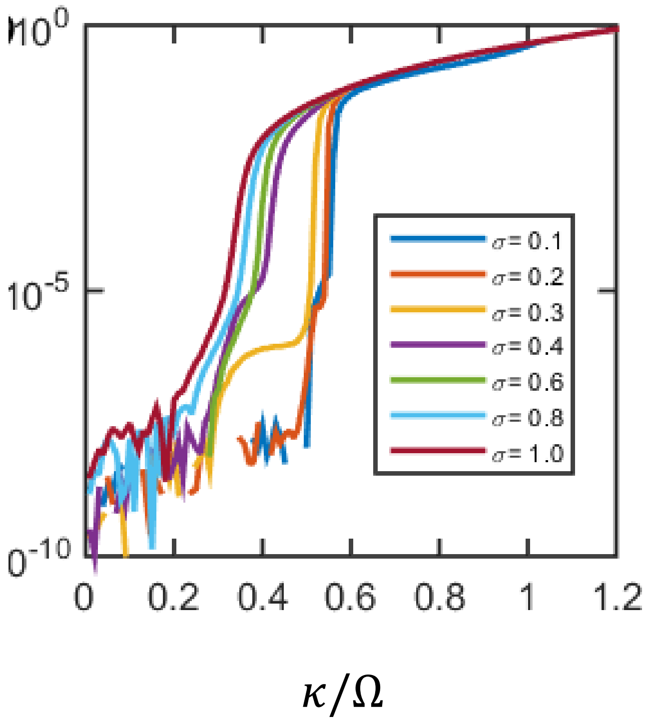
$$H^4 = -\frac{\kappa}{24}\sum_{j=1}^{N}(\phi_j - \phi_{j+1})^4 \sum_{m=1}^{\infty} \cos\left(\frac{2\pi m}{\tau}t\right),$$

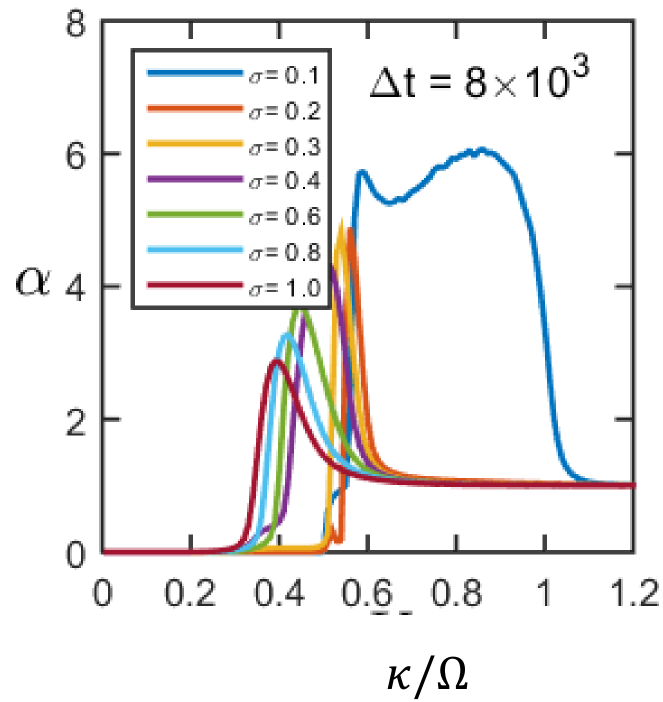
$$2\pi m \leq 4\cos^{-1}(1 \; -2\kappa/\Omega)$$

$$m = 4 \;\; \Rightarrow \;\; \frac{\kappa}{\Omega} > 0.5$$

Coupled kicked rotors – from quadratic to marginal

Initial state: $\langle \phi_j^2 \rangle = \sigma$





Finite size effect