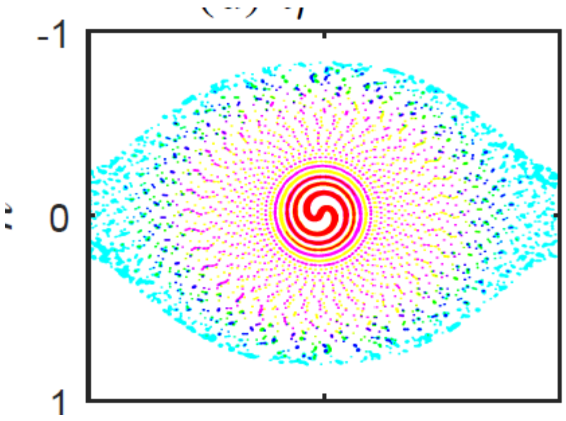
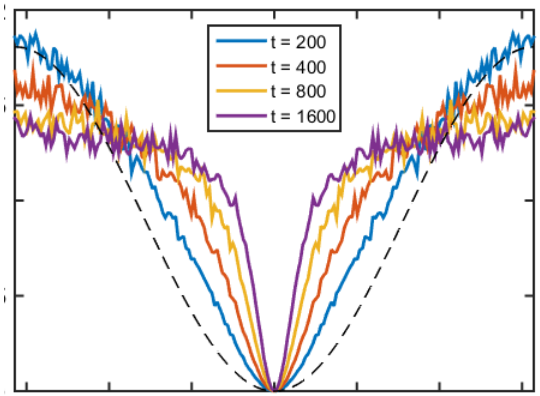
# Quantum-inspired prethermalization in classical systems




Bar-Ilan University
**Emanuele Dalla Torre**
http://www.nonequilibrium.org
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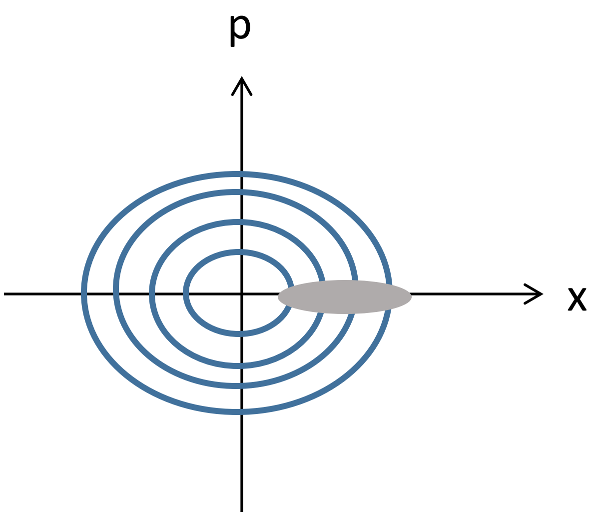
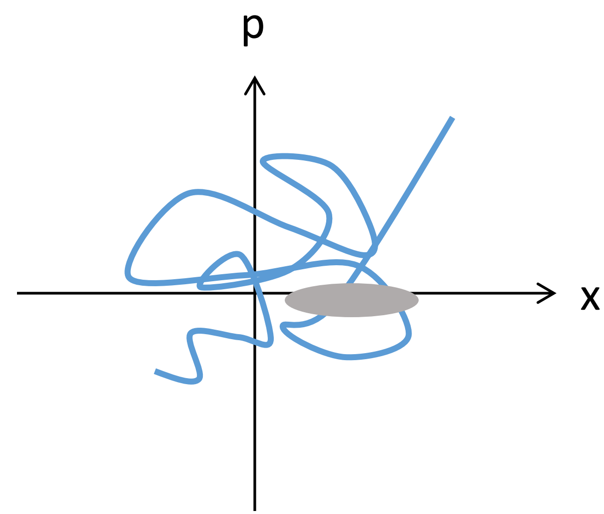
# Classical dynamics




Regular dynamics




Chaotic dynamics

Quantum inspired = evolution of an initial ensemble

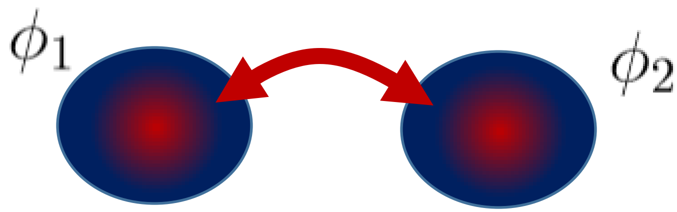
# Outline

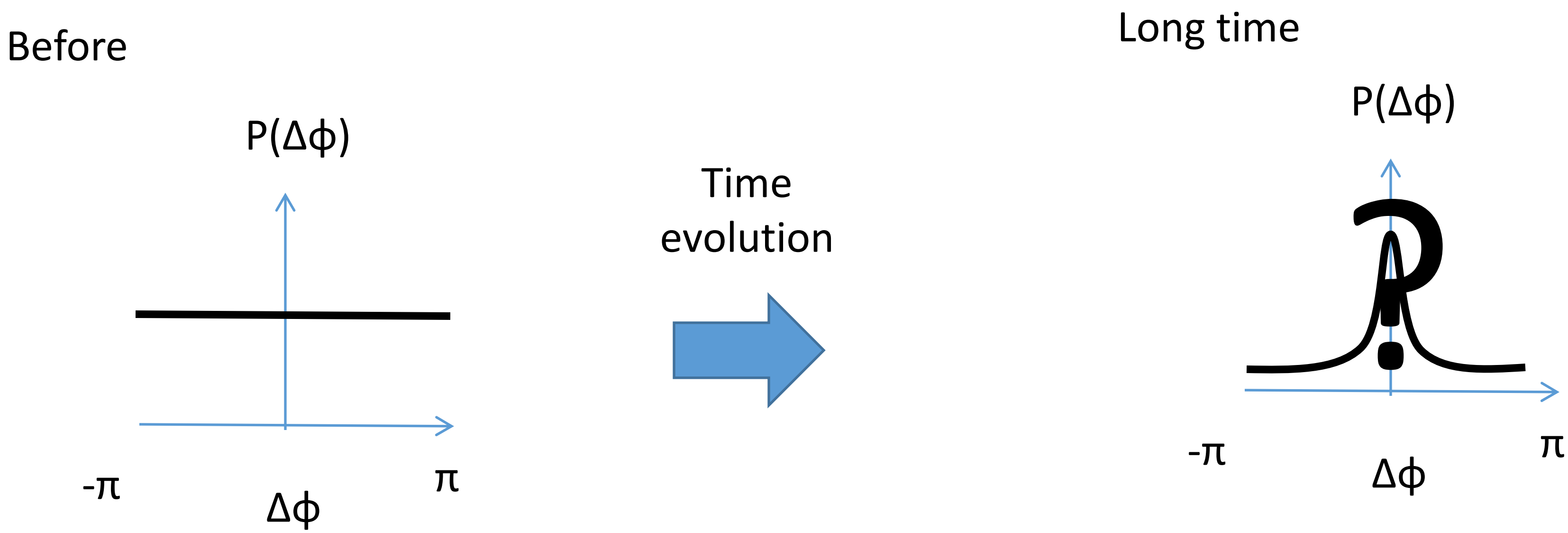
1. Few degrees of freedom (5 slides)

2. Many degrees of freedom (7 slides)

3. Extra (1 slide)

# Bosonic Josephson junctions

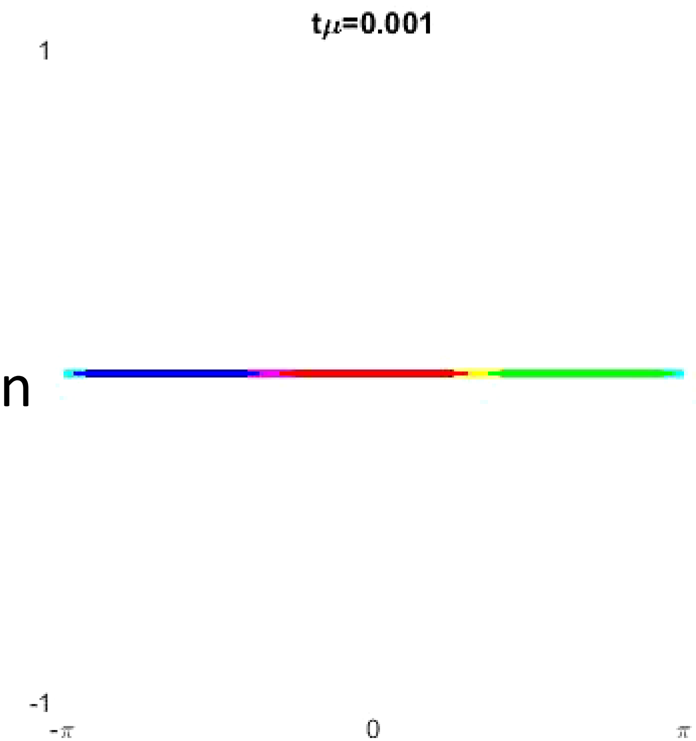


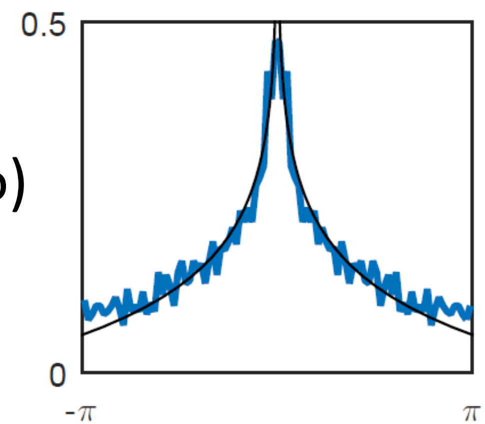




Dalla Torre, Demler, Plokovnikov (PRL, 2013)

# Bosonic Josephson junctions: time evolution

$$H = \frac{\mu}{2}\Delta n^2 + J\cos(\Delta\phi)$$



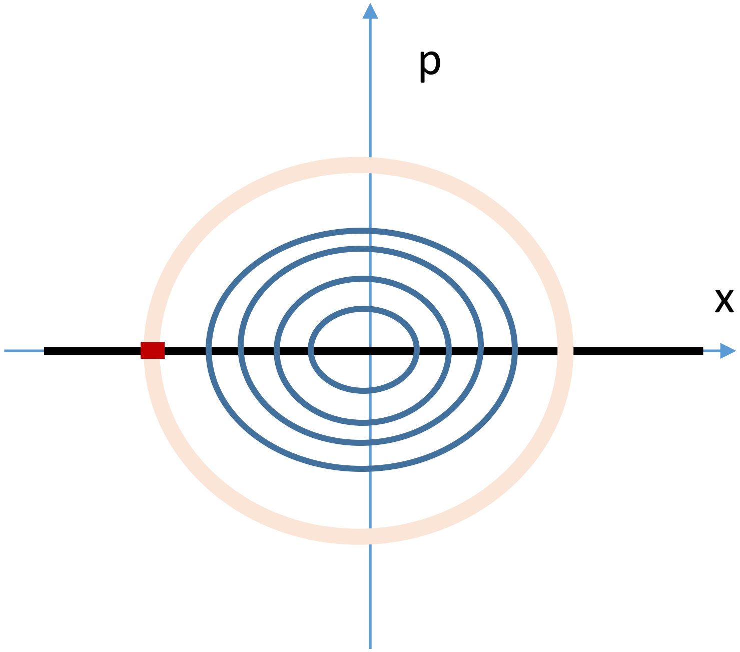




$$P(\Delta\phi) = -\frac{1}{\pi^2}\log(\Delta\phi)$$

# Scale invariant distribution functions

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega_0^2 x^2$$

$$\text{“}|\psi_0\rangle\text{”} = |p = 0\rangle$$




Before:

$$P(x,p) = A\delta(p)$$

$$P(x) = A$$

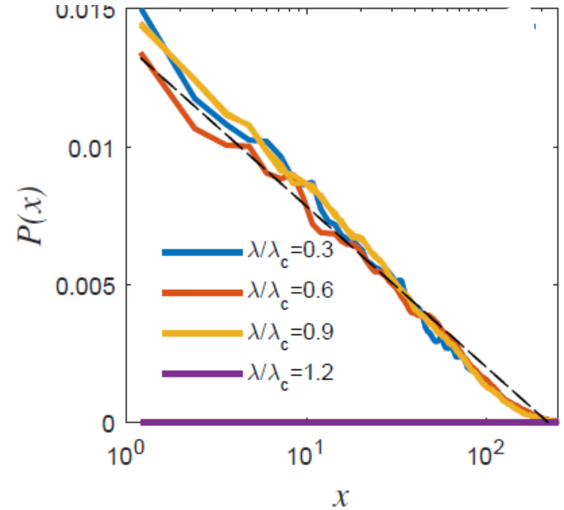
Long times:

$$P(x,p) = \frac{A\Delta x}{2\pi\sqrt{x^2+p^2}\Delta x}$$

$$P(x) = \frac{P_0}{\pi}\int_{-R}^{R} dp\, \frac{1}{\sqrt{x^2+p^2}} \longrightarrow -\frac{2P_0}{\pi}\log(|x|)$$

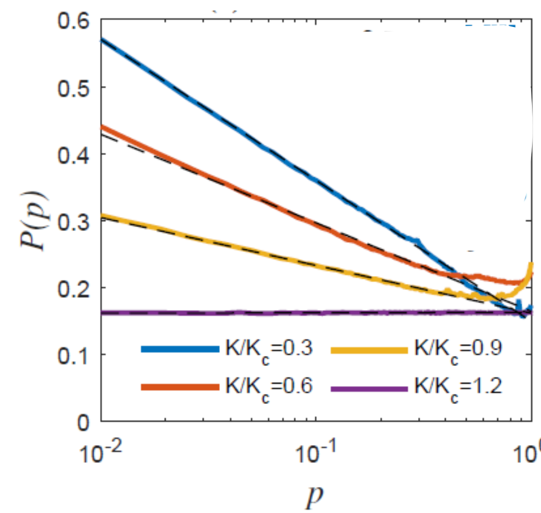
# Universal distribution function

$$P(\phi) = \kappa \log(\phi)$$




## Dicke model (semiclassical)

$$H = \frac{1}{2}(x^2 + p^2) + S_z + \lambda x S_x$$




## Kicked rotor

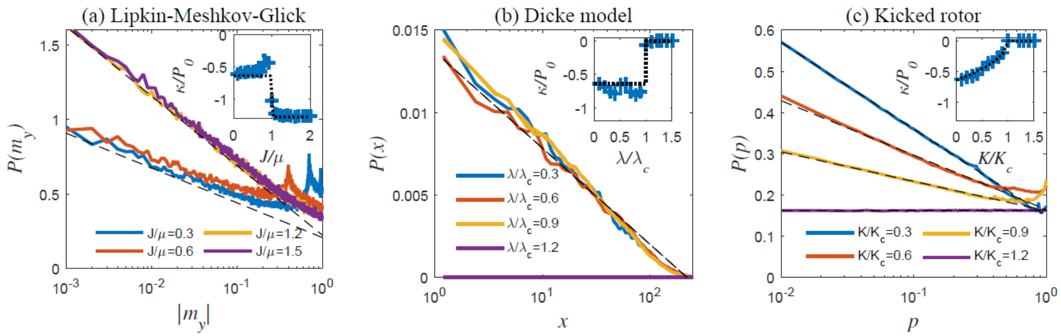
$$H(t) = \frac{1}{2}p^2 + \Delta(t)\cos(\phi)$$

$$\Delta(t) = \sum_n \delta(t - nT)$$

# Summary 1

Scale invariant distribution functions $$P(\phi) = \kappa \log(\phi)$$

are universal

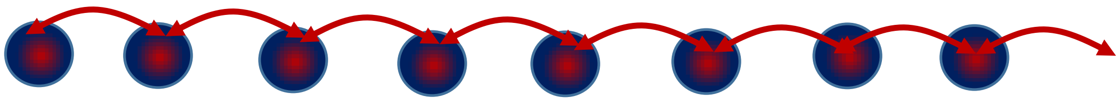



and distinguish regular (integrable) from chaotic (thermal)

$$P_{\text{thermal}} = Ae^{-E(\phi)/T} \approx Ae^{-\phi^2/\phi_0^2}$$

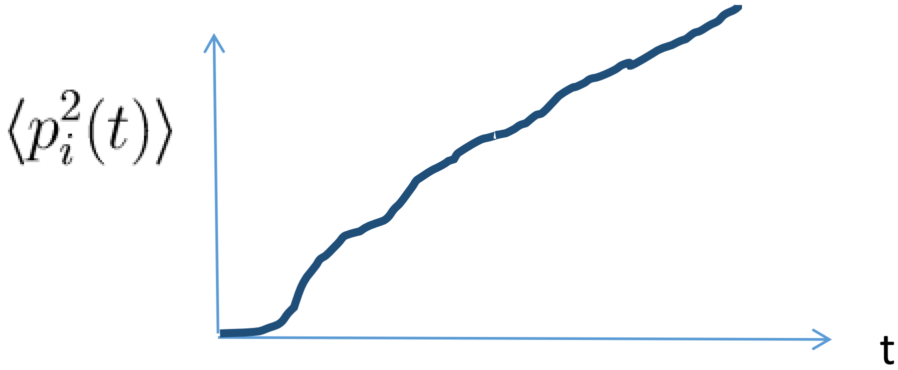
E. G. Dalla Torre, arxiv: 1709.01942

# Part 2 – many-body system



Many-body kicked rotor $$H(t) = \sum_i \frac{p_i^2}{2} + K\Delta(t)\cos(\phi_i - \phi_{i+1})$$

**Diffusion = heating**




How to prevent heating?

# How to prevent heating?

Quantum systems (Floquet theory)

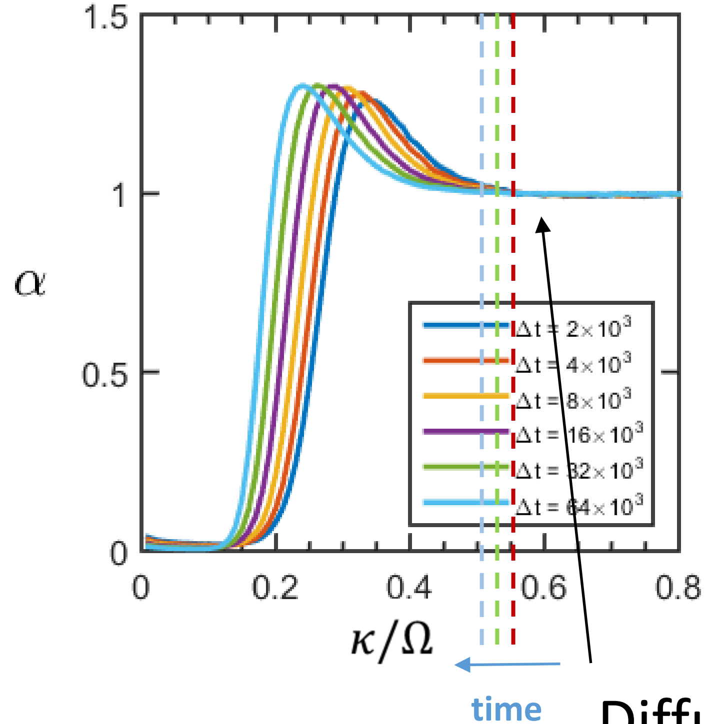
- Integrability
- Disorder (MBL)
- **High-frequency drive**

$$T \sim \exp(\Omega)$$

Choudhury & Mueller PRA 2014 - Bukov *etal* PRL 2015 - Abanin *etal* PRL 2015 – Goldman *etal* PRA 2015 - Chandra & Sondhi PRB 2016 – Canovi PRE 2016 - Abanin *etal* CMP 2017 - Lellouch *etal* PRX 2017 - Kuwahara *etal* Annals 2017 - Weidinger & Knap SR 2017 – Zeng & Sheng PRB 2017 – Else *etal* PRX 2017 - Abanin etal PRB 2017 -

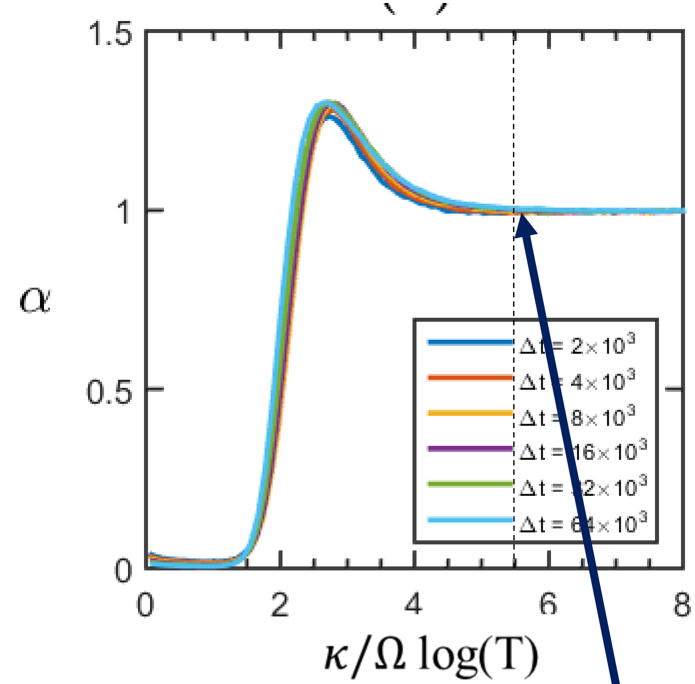
# Suppressed heating in coupled kicked rotors

$$\langle p_i^2(t)\rangle = At^{\alpha}$$




Diffusive

α=1




$$\frac{K}{\Omega}\log(T) = 5.5$$

# Suppressed heating in classical systems

$$H(t) = \sum_i \frac{p_i^2}{2} + K\Delta(t)\cos(\phi_i - \phi_{i+1}) \qquad \Delta(t) = \sum_n \delta(t - n\frac{2\pi}{\Omega})$$

Diffusion threshold $$\frac{K}{\Omega}\log(T^*) = 5.5 \Rightarrow T^* = \exp\left(5.5\frac{\Omega}{K}\right)$$

$$\Omega \to \infty$$

**High-frequency suppression of heating (Floquet Hamiltonian)**

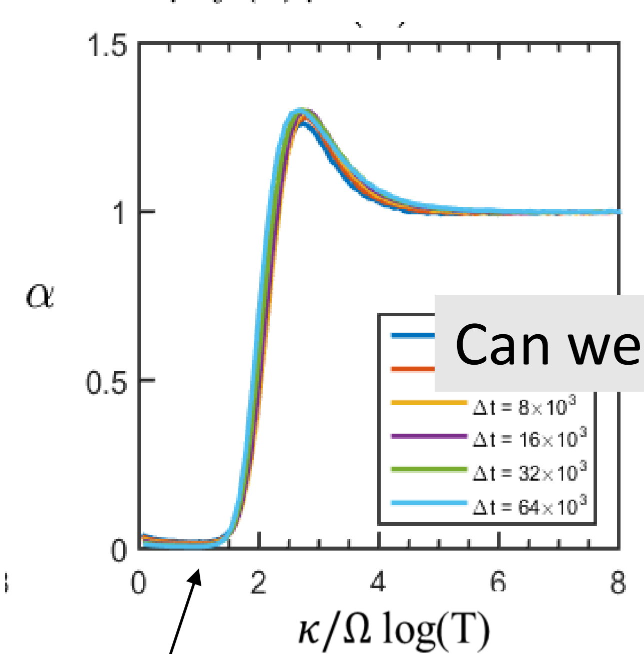
$$K \to 0$$

**Reduced diffusion close to integrability (Arnold diffusion)**

**Intro: Chirikov (1979)**

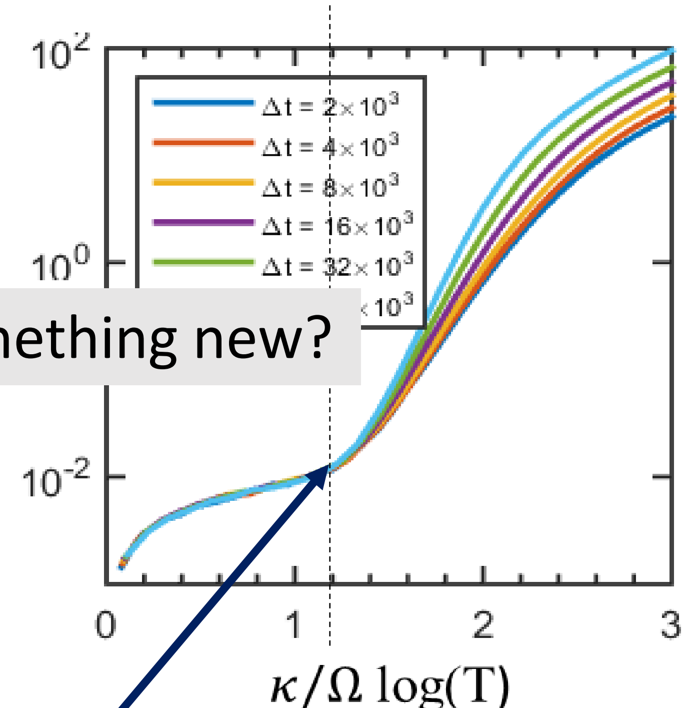
# Localization in classical systems

$$\langle p_i^2(t)\rangle = At^\alpha$$




Localized?

Can we learn something new?



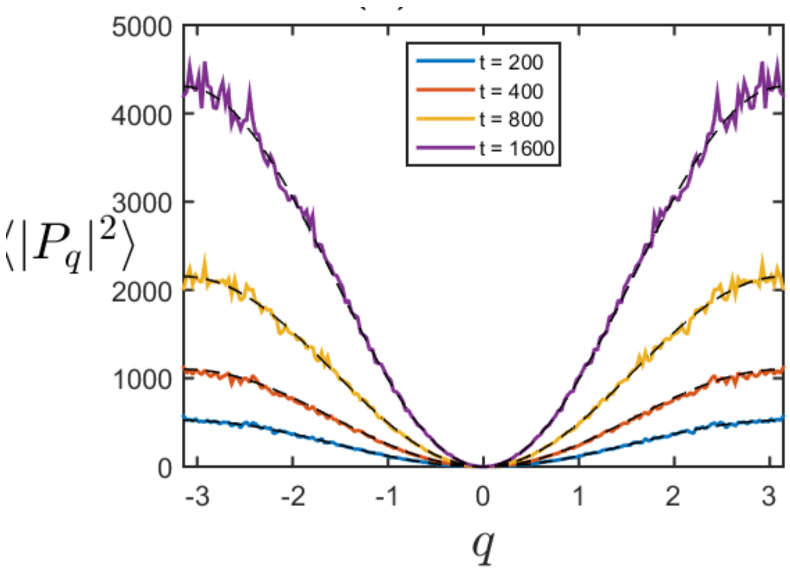

$$\frac{\kappa}{\Omega}\log(T) = 1.2$$

$$T^{**} = \exp\left(1.2\frac{\Omega}{K}\right)$$

# Prethermalization in classical systems

## Diffusive

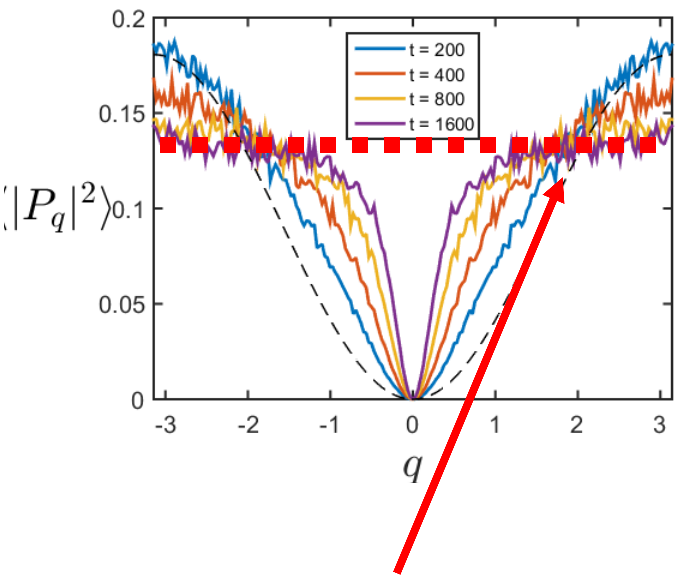
$$t > T^* \qquad \langle p_i^2(t) \rangle = At$$




$$P_q = \sum_n e^{iqn} \phi_n$$

## Localized

$$t < T^{**} \qquad \langle p_i^2(t) \rangle = \text{const}$$




**Prethermalization**

# Prethermalization in classical systems

# Summary 2

suppression of heating in many-body classical systems

$$T^* = \exp\left(5.5\frac{\Omega}{K}\right)$$

QUANTUM:
**high-frequency limit**
(Magnus Expansion of the Floquet Hamiltonian)

↔

CLASSICAL:
classical systems close to **integrability**
(Arnold diffusion)

NEW: prethermalization in classical systems

A. Rajak, R. Citro, E. G. Dalla Torre, arxiv:1801.01142
See also: Howell, Weinberg, Sels, Polkovnikov, Marin Bukov, arXiv:1802.04910
See also: Takashi Mori, arXiv:1804.02165

# How, who, and where?

$$ **Israel Science Foundation**,
Allon Fellowship

**Bar-Ilan University**

- Emanuele Dalla Torre
- **<u>Dr. Angelo Russomanno (→ ICTP, Trieste)</u>**
- Rajeev Singh (→ IIT, India)
- Bat-El Friedman
- David Dentelski
- Mor Roses
- **<u>Dr. Atanu Rajak</u>**
- Leon Bello

**University of Salerno**

- **<u>Roberta Citro</u>**

# Non-equilibrium Many-Body Quantum Physics

**Periodically-driven systems**

Dr. Atanu Rajak:
Prethermalization of
classical systems

**Topology in one dimension**

Bat-El Friedman:
String order and
duality transformations

**Dynamics of closed systems**

Maty Aharonian:
colliding condensates
(Landau criterion)

**Superconducting devices**

Prof. Yachin Ivry (Technion):
Multiport SQUID
device (exp)
[APL]

**Quantum optics**

Mor Roses:
driven-dissipative
Dicke model

Dr. Marcello Strinati:
Optical Parametric
Oscillators (th+exp)

**Strongly correlated materials**

David Dentelski:
superconducting fluctuations
In disordered materials [PRB]

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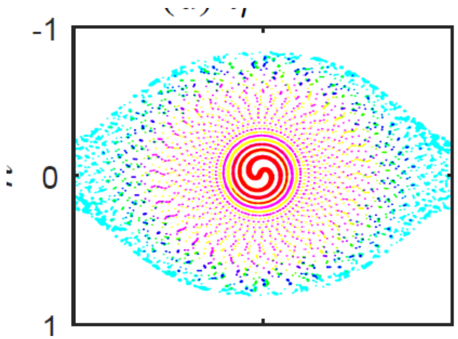
# Quantum-inspired prethermalization in classical systems

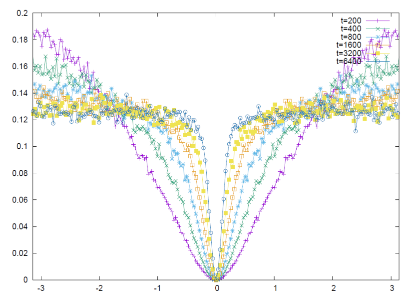
1. FEW BODY: Universal scale invariant distribution functions




$$P(\phi) = \kappa \log(\phi)$$

arxiv: 1709.01942

2. MANY BODY: Prethermalization in classical many-body systems

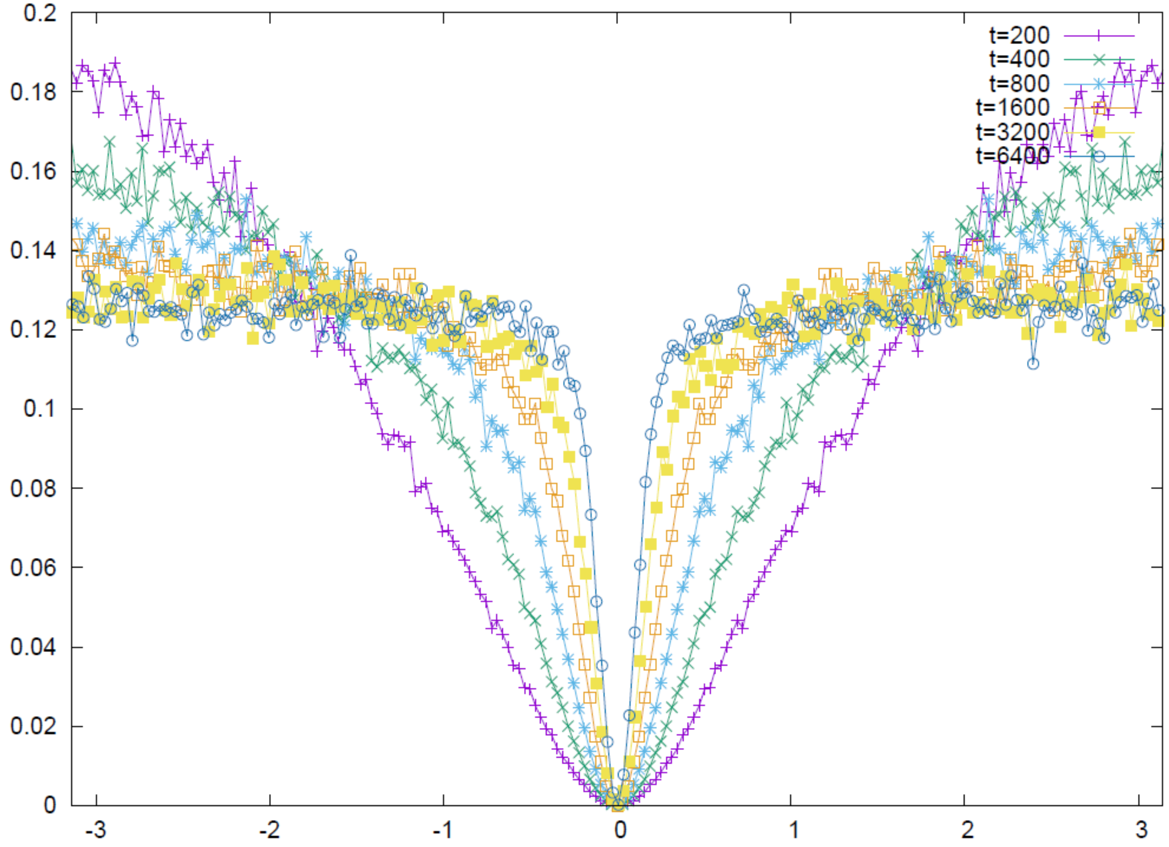



$$T^* = \exp\left(5.5\frac{\Omega}{K}\right)$$

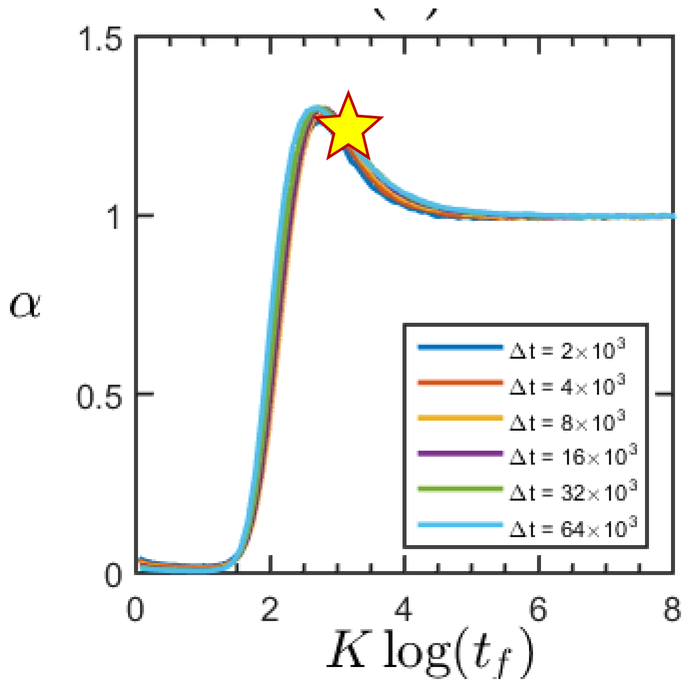
arxiv:1801.01142

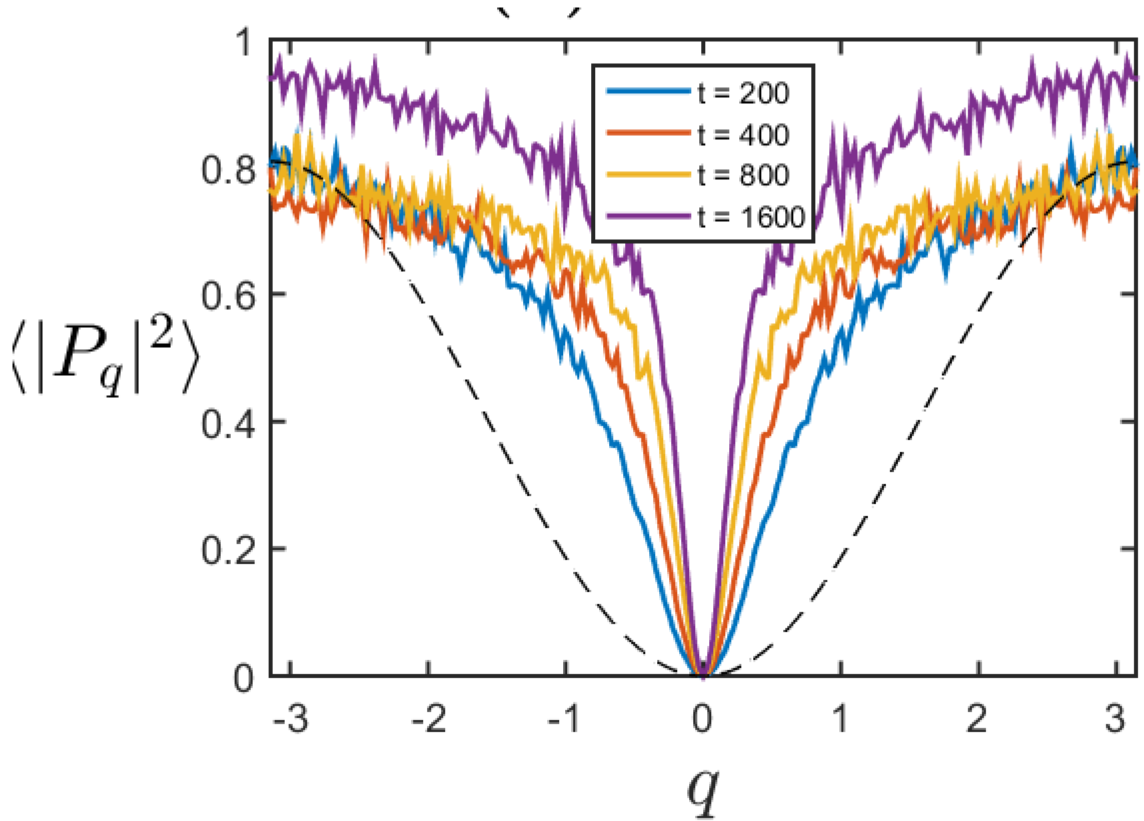
# Extra slides

# Prethermalization (longer times)



# Super-diffusion



# Finite size effect