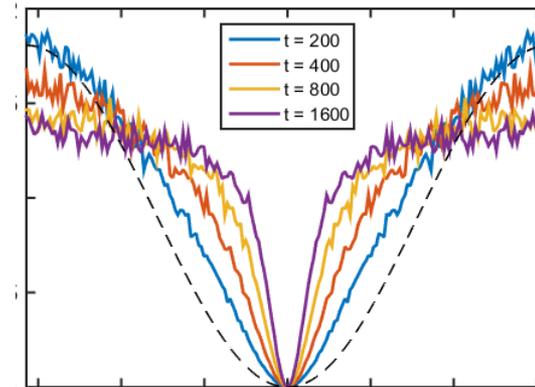
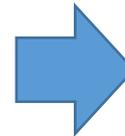
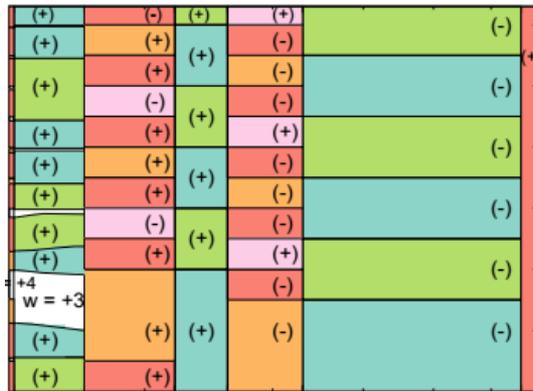


Emanuele Dalla Torre

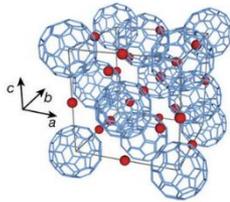
# From Floquet Engineering to Pre-thermalization



# Why Floquet Engineering ?

## Superconductors

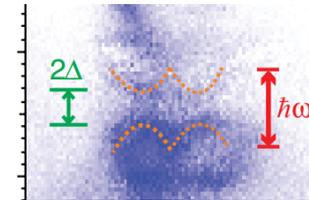
→ Light-enhanced superconductivity



[Cavalleri group – Nature 2016 ]

## Topological insulators

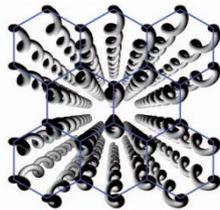
→ Floquet band structure



[Gedik group – Science 2013]

## Wave guides

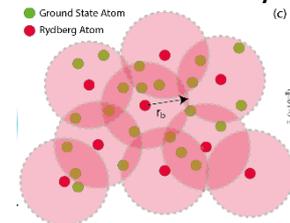
→ Photonic topological insulators



[Segev-Szameit group – Science 2018]

## Rydberg atoms

→ Discrete time crystals



[Lukin group – Nature 2017]

# Floquet Engineering - idea

Periodically driven Hamiltonian

$$H(t) = H(t + T)$$



Stroboscopic time evolution

$$|\psi(nT)\rangle = (U(T))^n |\psi_0\rangle$$



Effective Floquet Hamiltonian

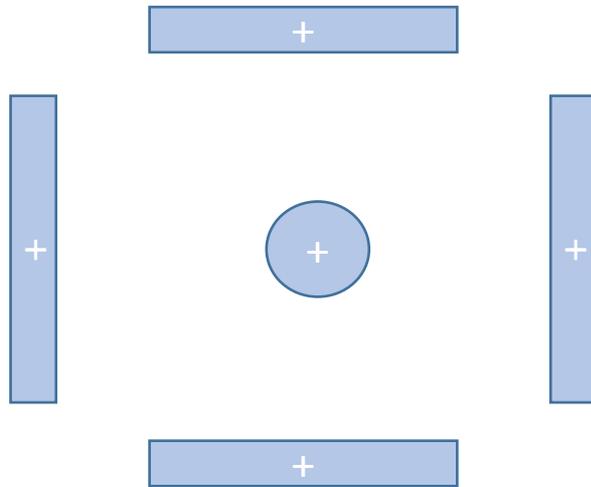
$$|\psi(nT)\rangle = e^{-i H_F n T} |\psi_0\rangle$$

$$U(T) = e^{-i H_F T}$$

$H_F$  is cooooool!

Eckardt, RMP 2017

# Example 1 – Pauling trap



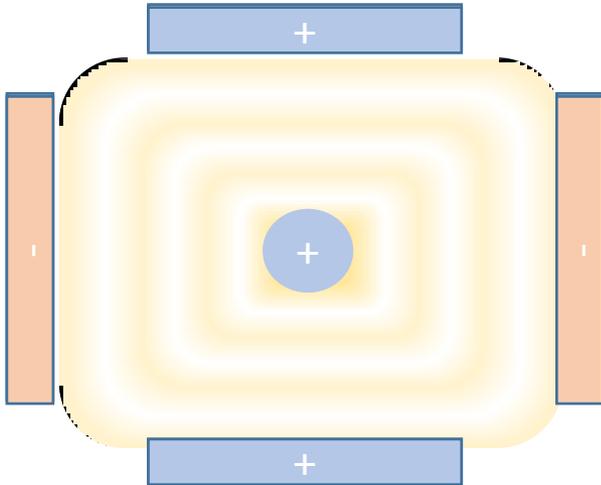
$$\text{Trapping : } \begin{cases} \vec{\nabla} \phi = 0 \\ \nabla^2 \phi > 0 \end{cases}$$

But...

$$\nabla^2 \phi = -\vec{\nabla} \cdot \vec{E} = -4\pi\rho = 0$$

→ No static trapping in vacuum

# Example 1 – Pauling trap



$$H(t) = \frac{p^2}{2m} + V(x^2 - y^2) \cos(\Omega t)$$

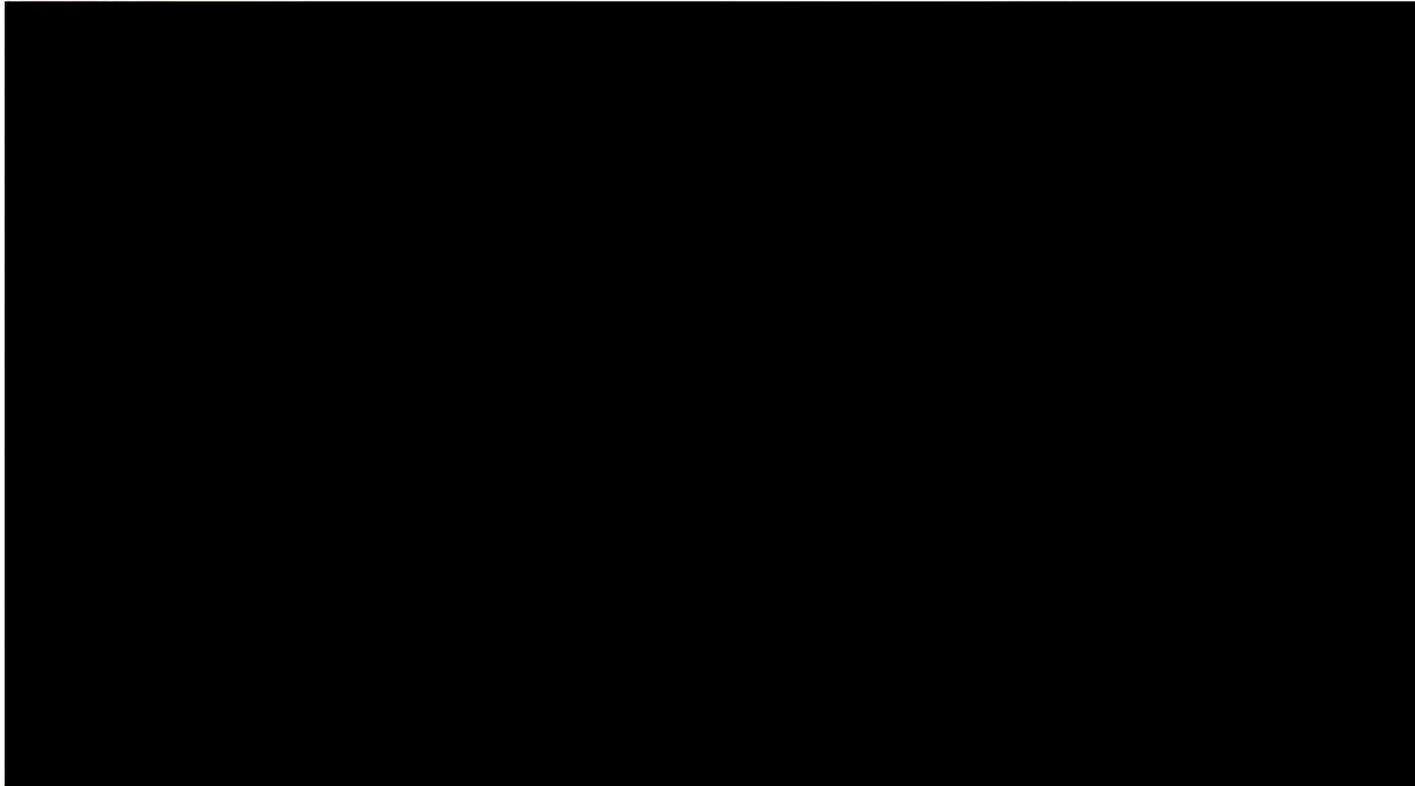
$$H_{av} = \frac{p^2}{2m}$$

Magnus expansion

$$H_F = H_{av} + \frac{1}{\Omega} \int dt \int dt' [H(t), H(t')] + \dots$$

$$H_F = \frac{p^2}{2m} + \frac{V^2}{\Omega^2 m} (x^2 + y^2)$$

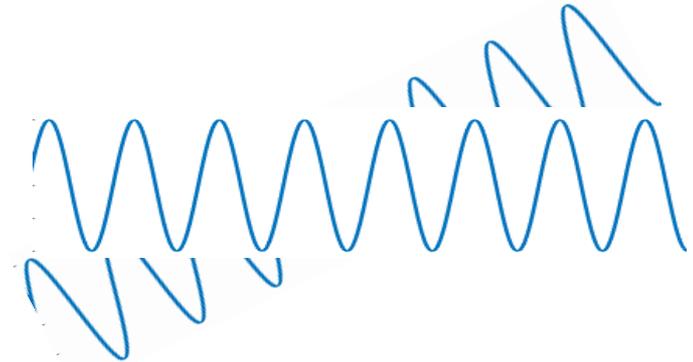
# Example 1 – Pauling trap



Video credit: [Harvard Natural Sciences Lecture Demonstrations](#)

## Example 2 – lattice shaking

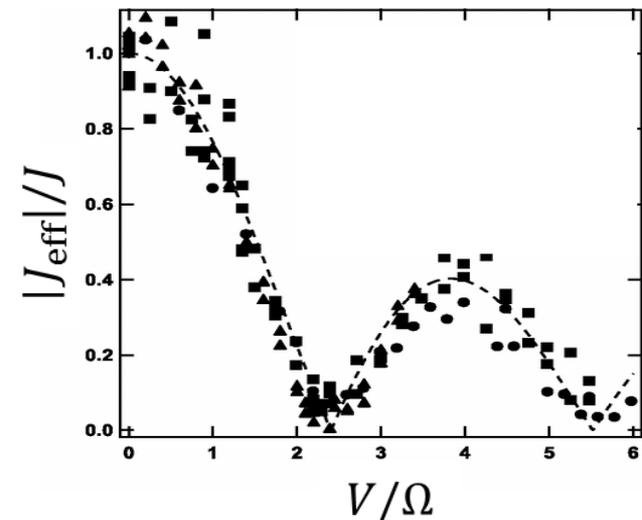
$$H(t) = \sum_i J(c_i^\dagger c_{i+1} + H.c.) + V \sum_i n_i \cos(\Omega t)$$



“Gauge transformation” (rotating frame)

$$H(t) = J \sum_i (e^{iV \cos(\Omega t)} c_i^\dagger c_{i+1} + H.c.)$$

$$H_F \approx H_{av} = \underbrace{J J_0 \left( \frac{V}{\Omega} \right)}_{J_{\text{eff}}} \sum_i (c_i^\dagger c_{i+1} + H.c.)$$



Morsch-Arimondo PRL 2007 & PRL 2008

# Outline

## Periodically driven spin chain

- **Periodically driven Ising model** → **Time crystal**

Russomanno, Dalla Torre (EPL 2016)

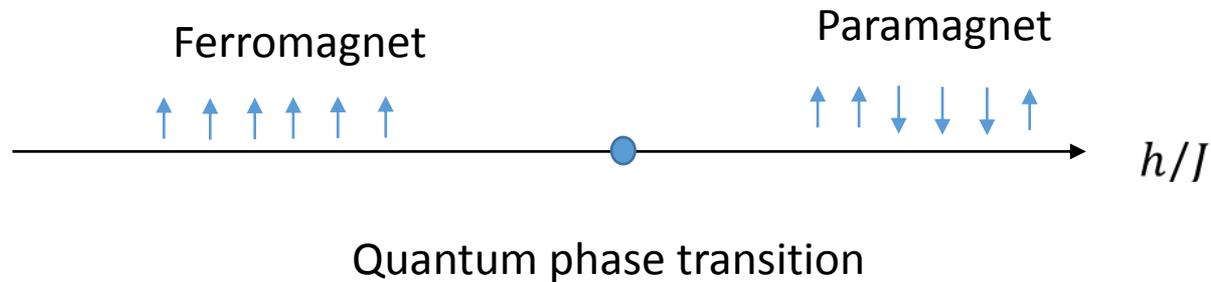
Russomanno, Friedman, Dalla Torre (PRB 2017)

- **Many body kicked rotor** → **Prethermalization**

Rajak, Citro, Dalla Torre (arXiv:1801.01142)

# Ising model : static

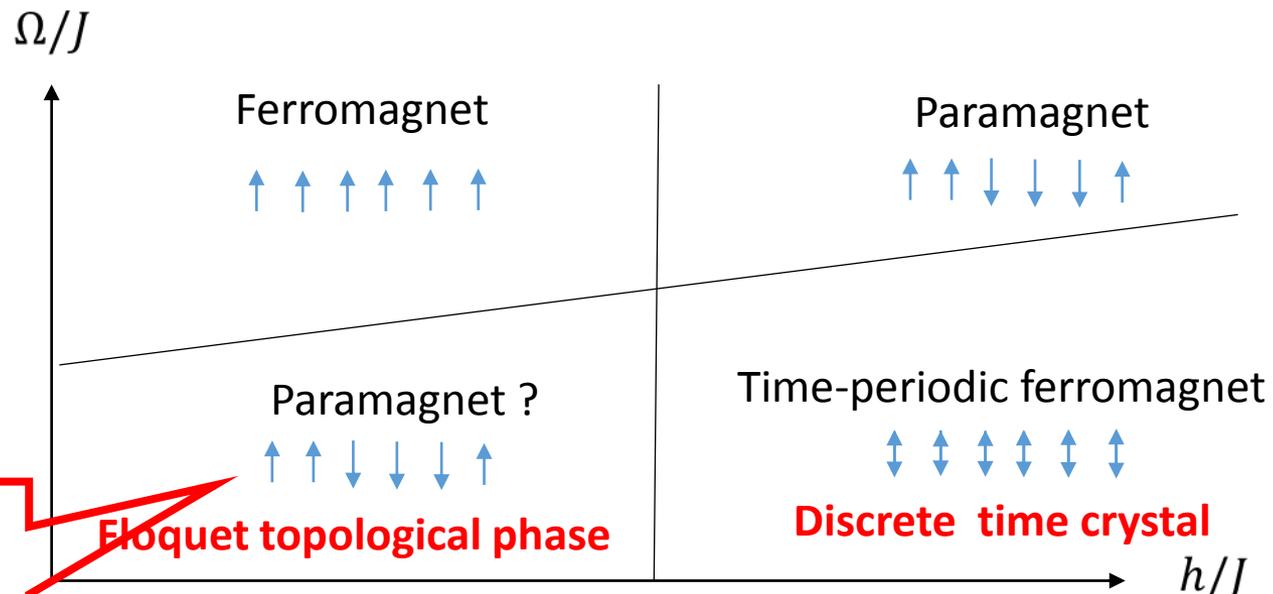
$$\hat{H}(t) = \sum_j (-J \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + h(t) \hat{\sigma}_j^x) \quad h(t) = h_0$$



# Ising model : periodic drive

$$\hat{H}(t) = \sum_j (-J \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + h(t) \hat{\sigma}_j^x)$$

$$h(t) = h_0 + \delta h \cos(\Omega t)$$



Non local order

$$\hat{\beta}_j^y = \left( \prod_{k=1}^{j-1} \hat{\sigma}_k^x \right) \hat{\sigma}_j^z \hat{\sigma}_{j+1}^y$$

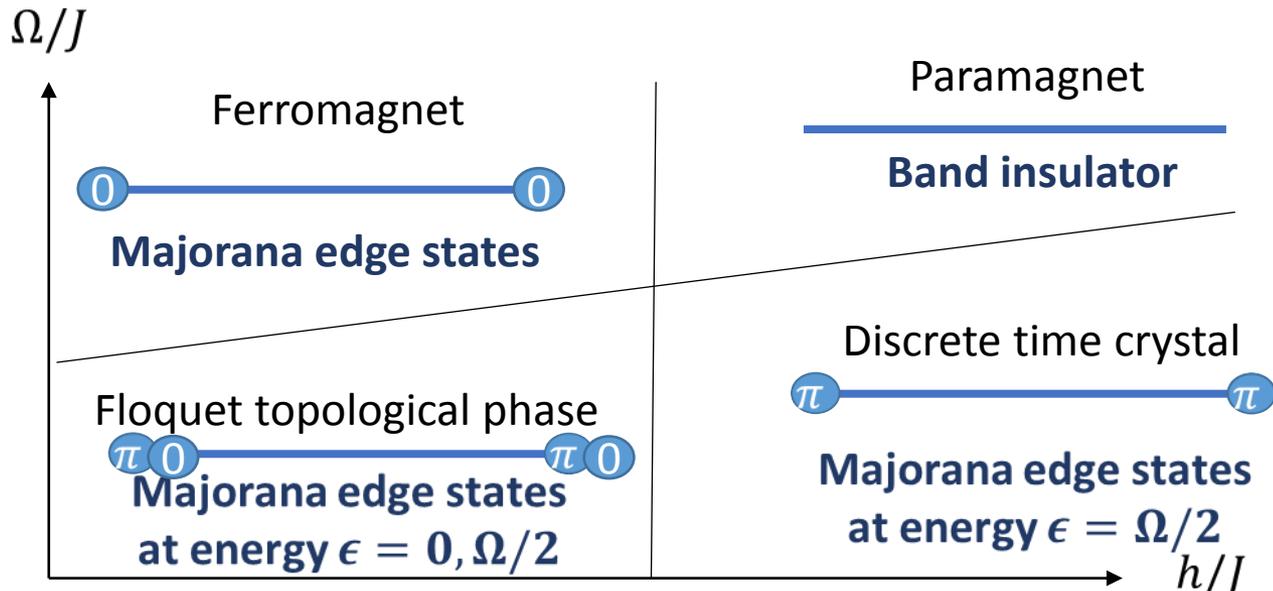
Else, Bauer, Nayak (PRL 2016)

Russomanno, Friedman, Dalla Torre (PRB 2017)

# Ising model : periodic drive

Jordan-Wigner transformation

$$\hat{H}(t) = \sum_j (-J \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + h(t) \hat{\sigma}_j^x) \quad \rightarrow \quad H = \sum_j J (c_i + c_i^\dagger)(c_{i+1} - c_{i+1}^\dagger) + h(t) c_i^\dagger c_i$$



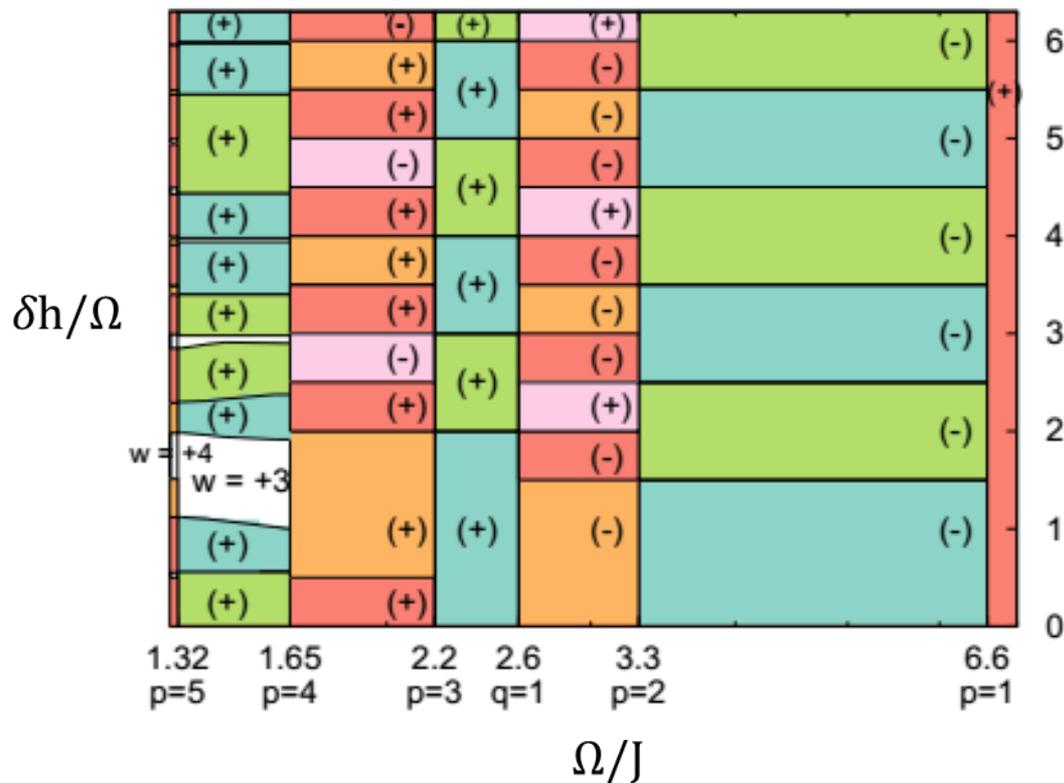
End of the story?

Else, Bauer, Nayak PRL 2016

# Ising model : periodic drive

$$\hat{H}(t) = \sum_j (-J \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + h(t) \hat{\sigma}_j^x)$$

$$h(t) = 2.3 J + \delta h \cos(\Omega t)$$



- Paramagnet
- Ferromagnet
- Time crystals
- Topological phases
- Floquet topological phase

Russomanno, Dalla Torre (EPL 2016)

Russomanno, Friedman, Dalla Torre (PRB 2017)

# The problem : heating

Break integrability = Add interactions



Thermalize (Eigenstate thermalization hypothesis)



Steady state = infinite temperature

$$Z = 1$$



# How to prevent heating?

- ✓ Integrability
- ✓ Disorder (many-body localization)
- ✓ **High-frequency drive (prethermalization)**

Classical many-body:

?

Quantum many-body:

Choudhury & Mueller PRA 2014 - Bukov *etal* PRL 2015  
- Abanin *etal* PRL 2015 – Goldman *etal* PRA 2015 -  
Chandra & Sondhi PRB 2016 – Canovi PRE 2016 -  
Abanin *etal* CMP 2017 - Lellouch *etal* PRX 2017 -  
Kuwahara *etal* Annals 2017 - Weidinger & Knap SR  
2017 – Zeng & Sheng PRB 2017 – Else *etal* PRX 2017 -  
Abanin *etal* PRB 2017.

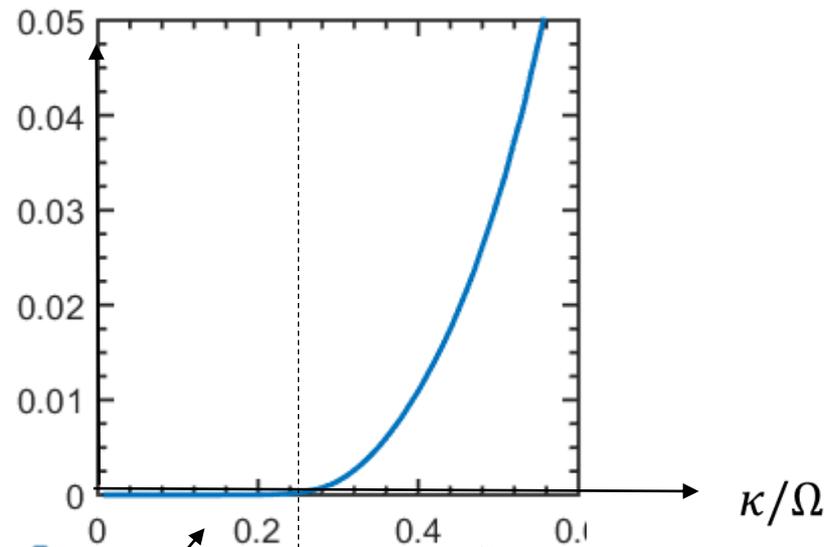
# Coupled Kicked rotors

$$H = \sum_{j=1}^N \left[ \frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right].$$

Always diffusive  
 Kaneko & Konishi (1989)  
 Chirikov & Vecheslavov (1997)

Diffusion coefficient

$$D = \frac{\langle p(T)^2 \rangle - \langle p(0)^2 \rangle}{T}$$

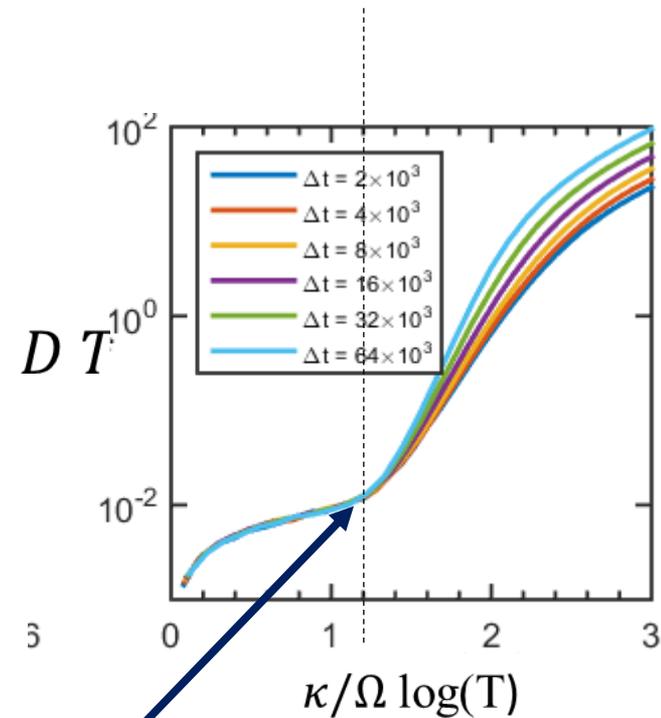
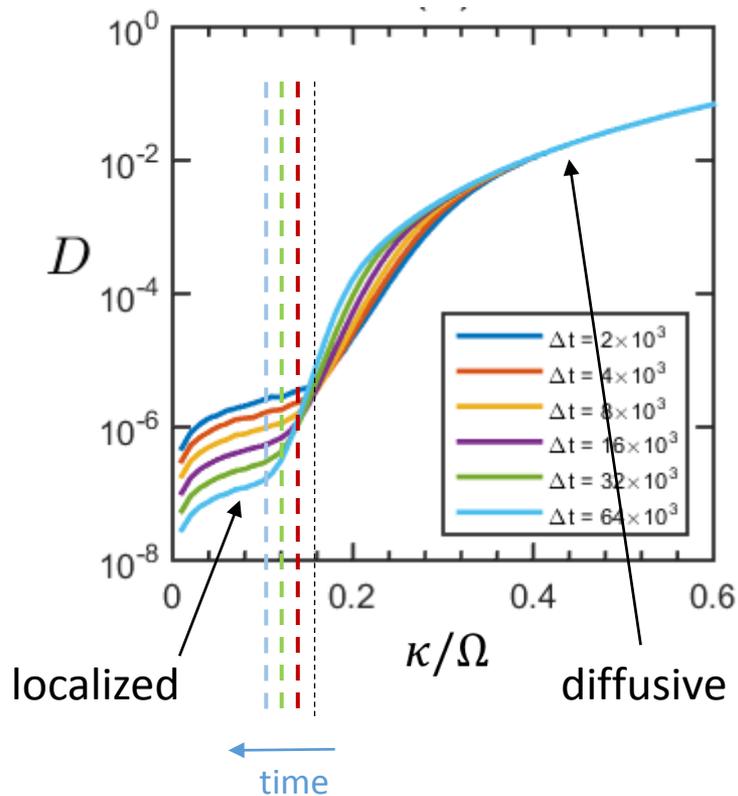


High frequency = localized

Low frequency = diffusive

# Coupled kicked rotors – scaling analysis

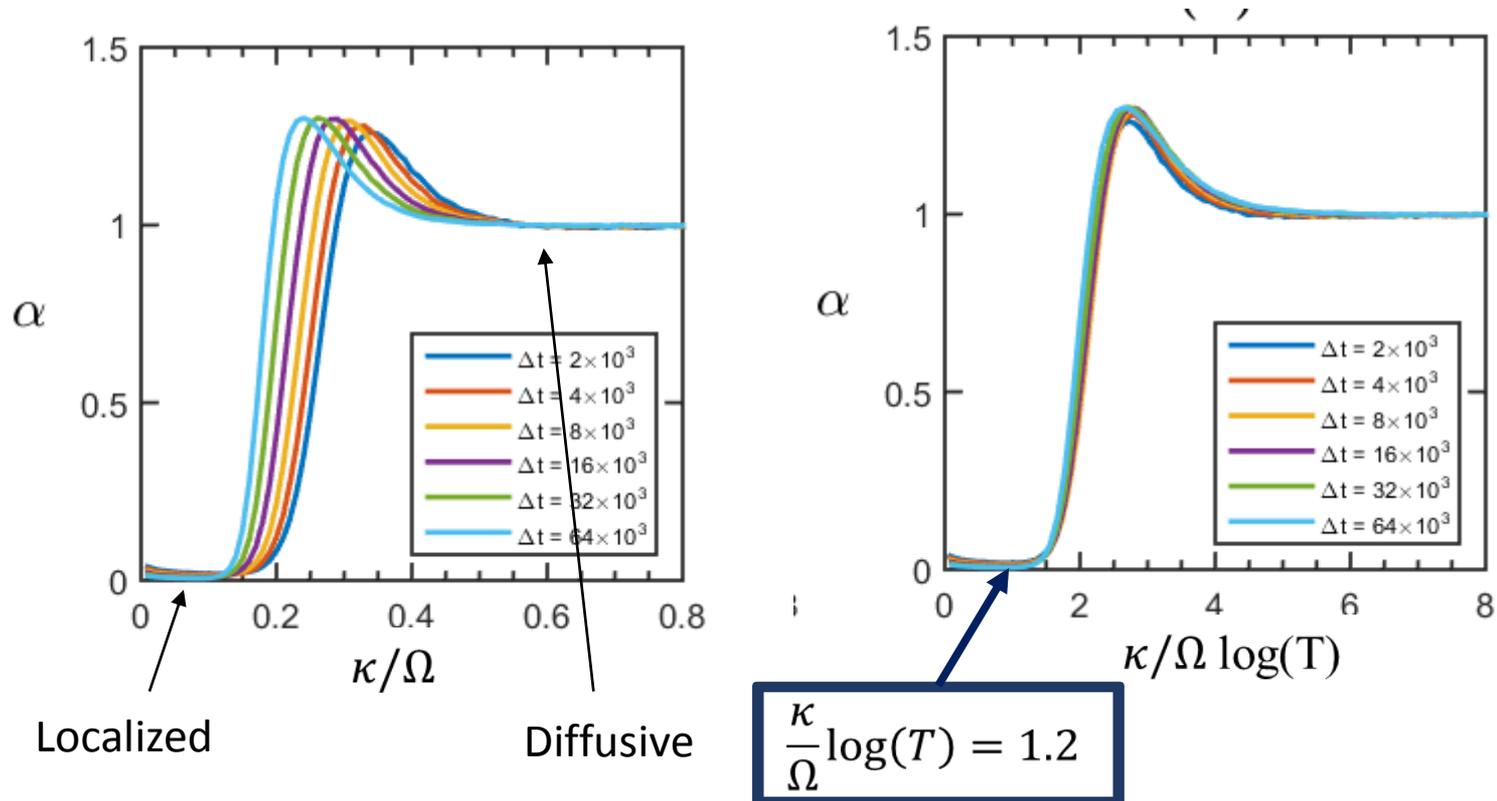
$$D = \frac{\langle p(T)^2 \rangle - \langle p(0)^2 \rangle}{T}$$



$$\frac{\kappa}{\Omega} \log(T) = 1.2$$

# Coupled kicked rotors – scaling analysis

$$\langle p(T)^2 \rangle = A t^\alpha$$

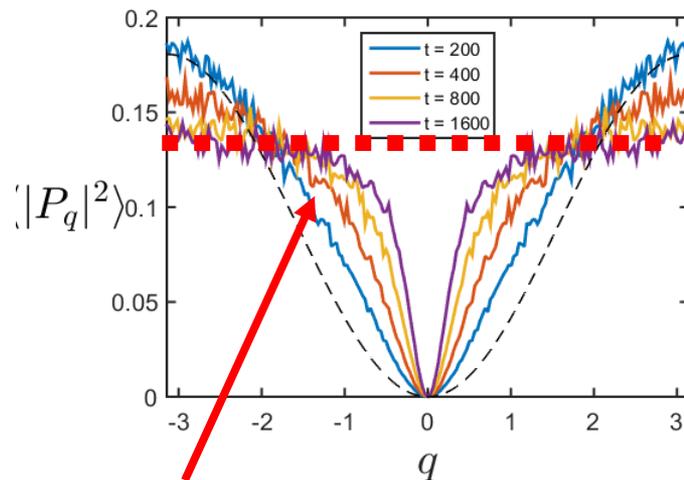


# “Marginal localization”

$$\frac{\kappa}{\Omega} \log(T^*) = 1.2 \quad \Rightarrow \quad T^* = \exp\left(\frac{\Omega}{\kappa}\right)$$

$$T \ll T^*$$

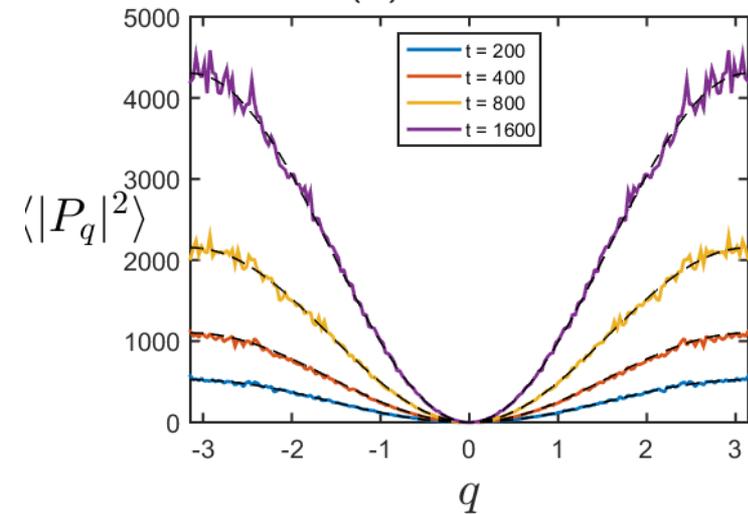
$$\langle P^2 \rangle = \text{const}$$



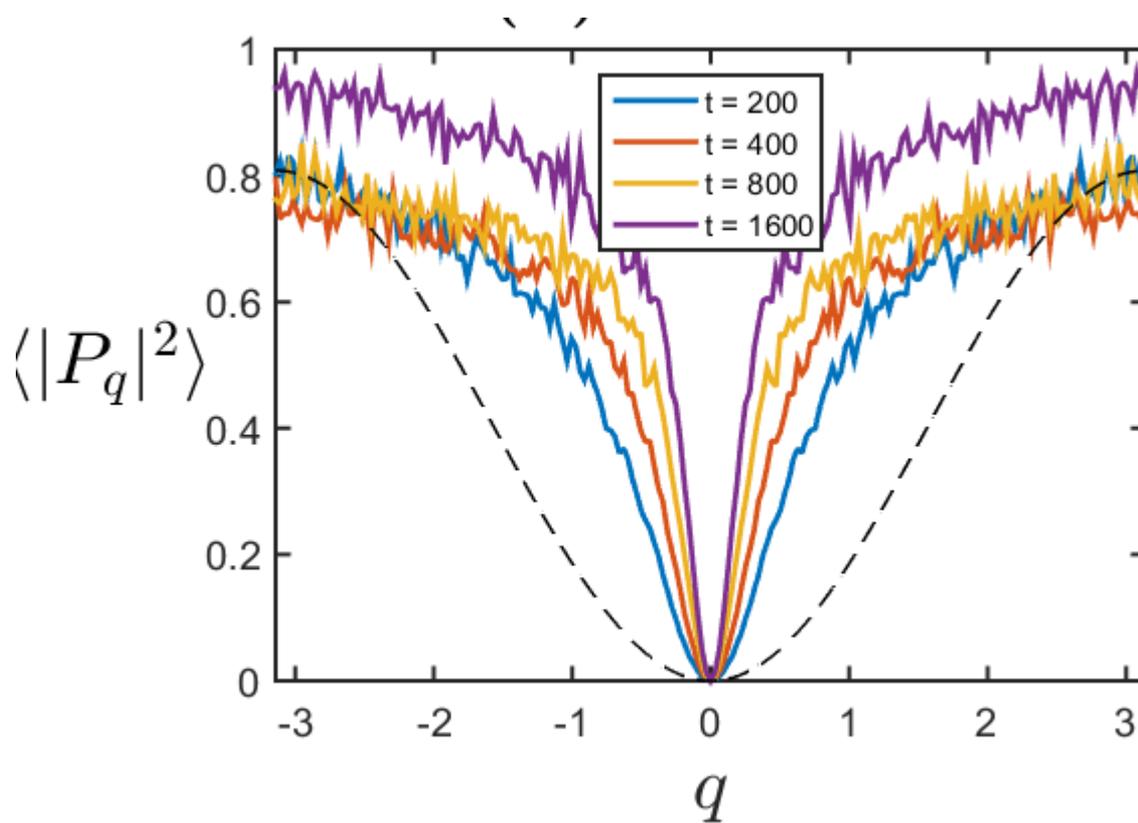
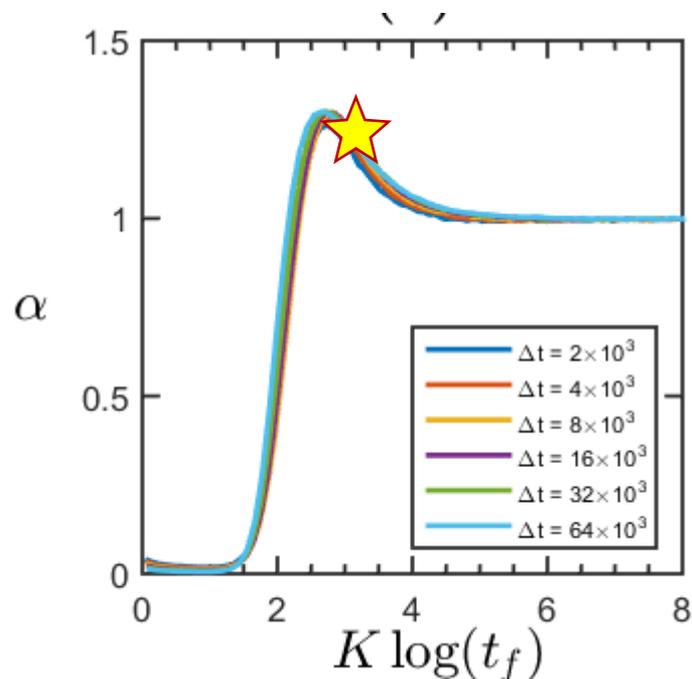
**Prethermalization**

$$T \gg T^*$$

$$\langle P^2 \rangle = D t$$



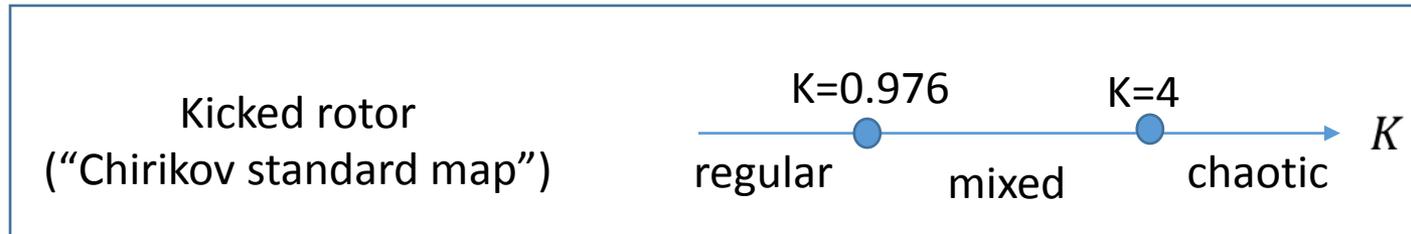
# Super-diffusion



# Coupled kicked rotors – quadratic expansion

$$H = \frac{1}{2} \sum_q \left[ |P_q|^2 + K(q) |\phi_q|^2 \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right],$$

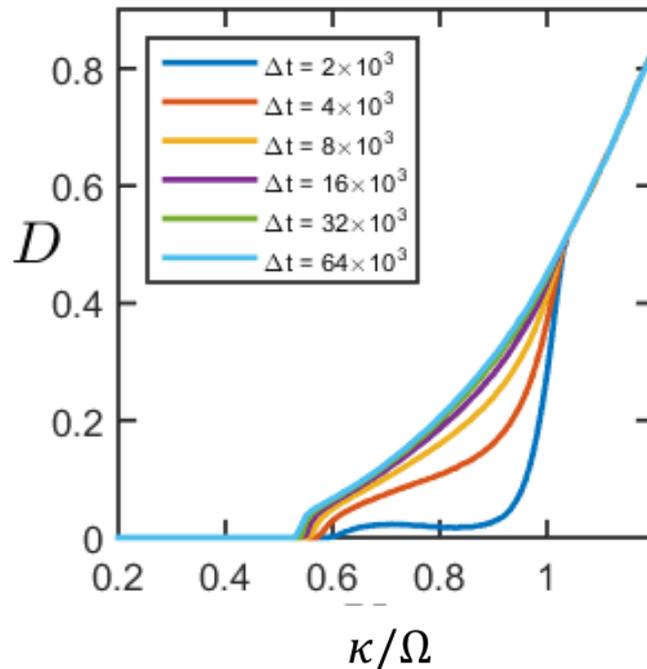
$$K(q) = \frac{4\kappa}{\Omega} \sin^2\left(\frac{q}{2}\right).$$



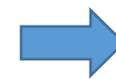
→ Transition at  $\kappa/\Omega = 1$

# Coupled kicked rotors – quadratic instability

Initial state:  $\phi_j \approx 0$



$$H^4 = -\frac{\kappa}{24} \sum_{j=1}^N (\phi_j - \phi_{j+1})^4 \sum_{m=1}^{\infty} \cos\left(\frac{2\pi m}{\tau} t\right),$$

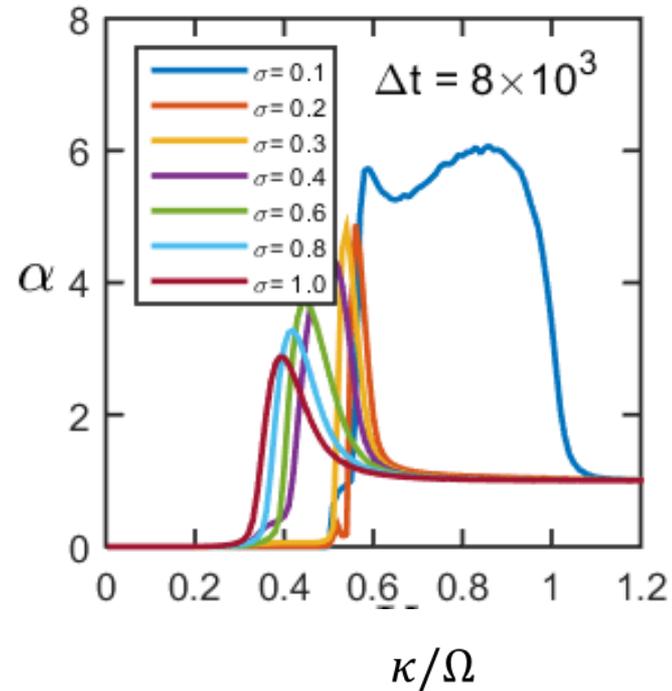
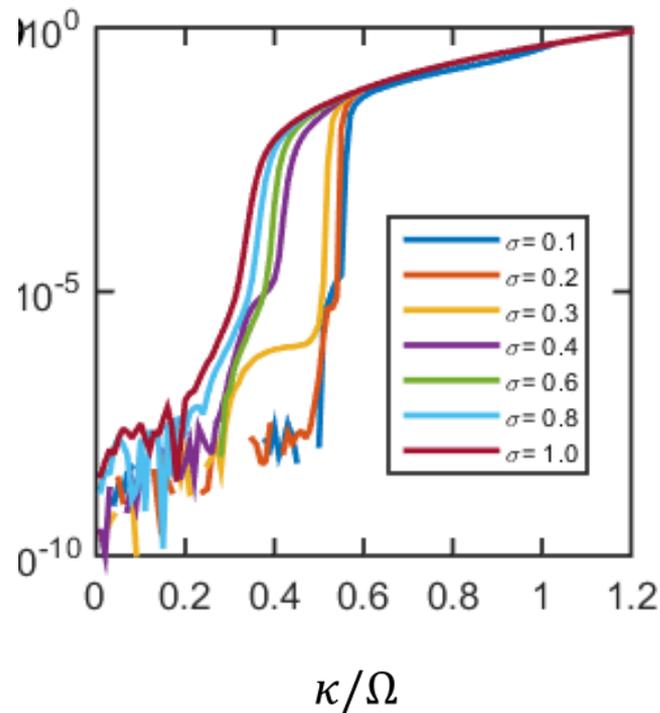


$$2\pi m \leq 4 \cos^{-1}(1 - 2\kappa/\Omega)$$

$$m = 4 \Rightarrow \frac{\kappa}{\Omega} > 0.5$$

# Coupled kicked rotors – from quadratic to marginal

Initial state:  $\langle \phi_j^2 \rangle = \sigma$



# From Floquet Engineering to Pre-thermalization

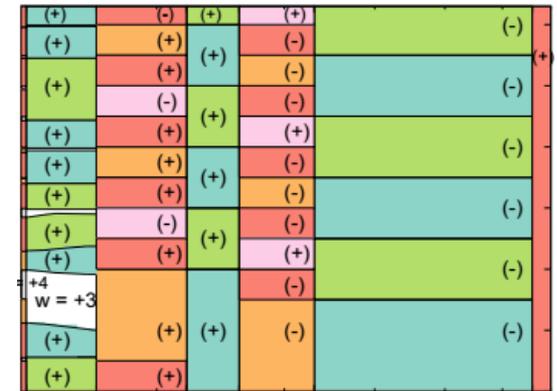
## Periodically driven spin chain

- Periodically driven *quantum* Ising model

→ Time crystal & Floquet topological phases

Russomanno, Dalla Torre (EPL 2016)

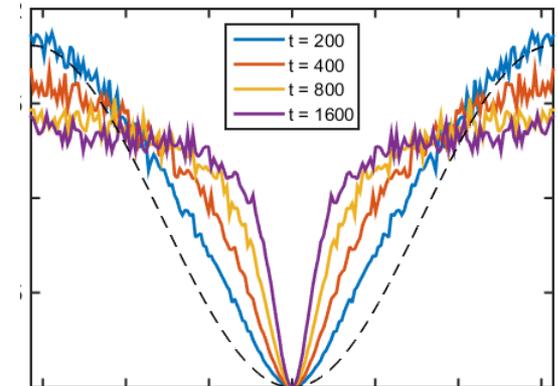
Russomanno, Friedman, Dalla Torre (PRB 2017)



- Many body *classical* kicked rotor

→ Localization & Prethermalization

Rajak, Citro, Dalla Torre (arXiv:1801.01142)



# How, who, and where?



\$\$ Israel Science Foundation,



## Booking receipt

Date: 23JUL2017

Mr. Emanuele Dalla Torre  
HATAMAR 2 DIRA 27  
5440202 Givat Shmuel  
Israel

Booking 0000546486  
Record locator: W2T69I  
Transaction ref: 170723101801636914  
Form of payment: Credit Card, MasterCard

### Booking details

Name	Ticket no.	Flight details	RBD/Fare basis	Price (EUR)
Mr. Emanuele Dalla Torre	2462106474570	ST1991/19FEB18	P/POW6M Tel Aviv - Hamburg	44.36
		ST1990/22FEB18	O/OOW6M Hamburg - Tel Aviv	0.00
				<b>44.36</b>

ROBERTO GILIO

## 1<sup>st</sup> Italy-Israel meeting on nonequilibrium Physics Bar-Ilan University (April 11-13, 2016)



**THE JERUSALEM POST**

**ITALIAN AND ISRAELI PHYSICISTS  
CONFER AT BAR-ILAN UNIVERSITY**

BY LIDAR GRAVÉ-LAZI / APRIL 13, 2016 06:05

"Italy is Israel's most important European partner in the field of science."

**SAVE THE DATE: 2<sup>nd</sup> meeting in Salerno (September 4-7, 2018)**

# Non-equilibrium Many-Body Quantum Physics



## Periodically-driven systems

Dr. Atanu Rajak:  
Dynamic localization of  
a many-body kicked rotor



## Topology in one dimension

Bat-El Friedman:  
String order and  
duality transformations



## Dynamics of closed systems

Maty Aharonian:  
colliding condensates  
(Landau criterion)



## Quantum optics

Mor Roses:  
Real-time dynamics  
of the extended Dicke model



## Strongly correlated materials

David Dentelski:  
Short vs. Long-range  
superconducting fluctuations



Dr. Marcello Strinati:  
Non-linear optics

# From Floquet Engineering to Pre-thermalization

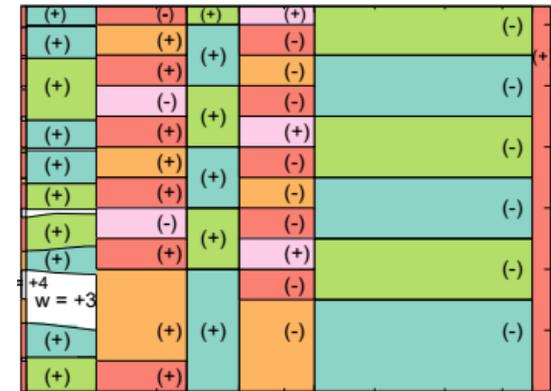
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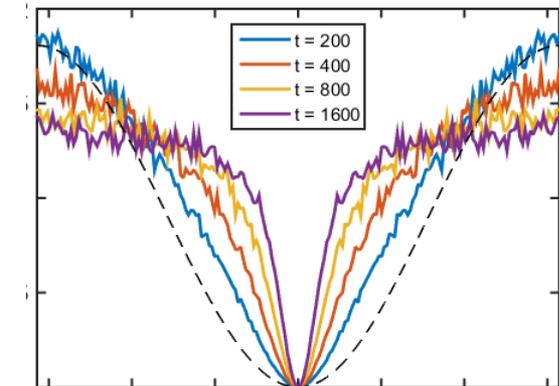
Russomanno, Friedman, Dalla Torre (PRB 2017)



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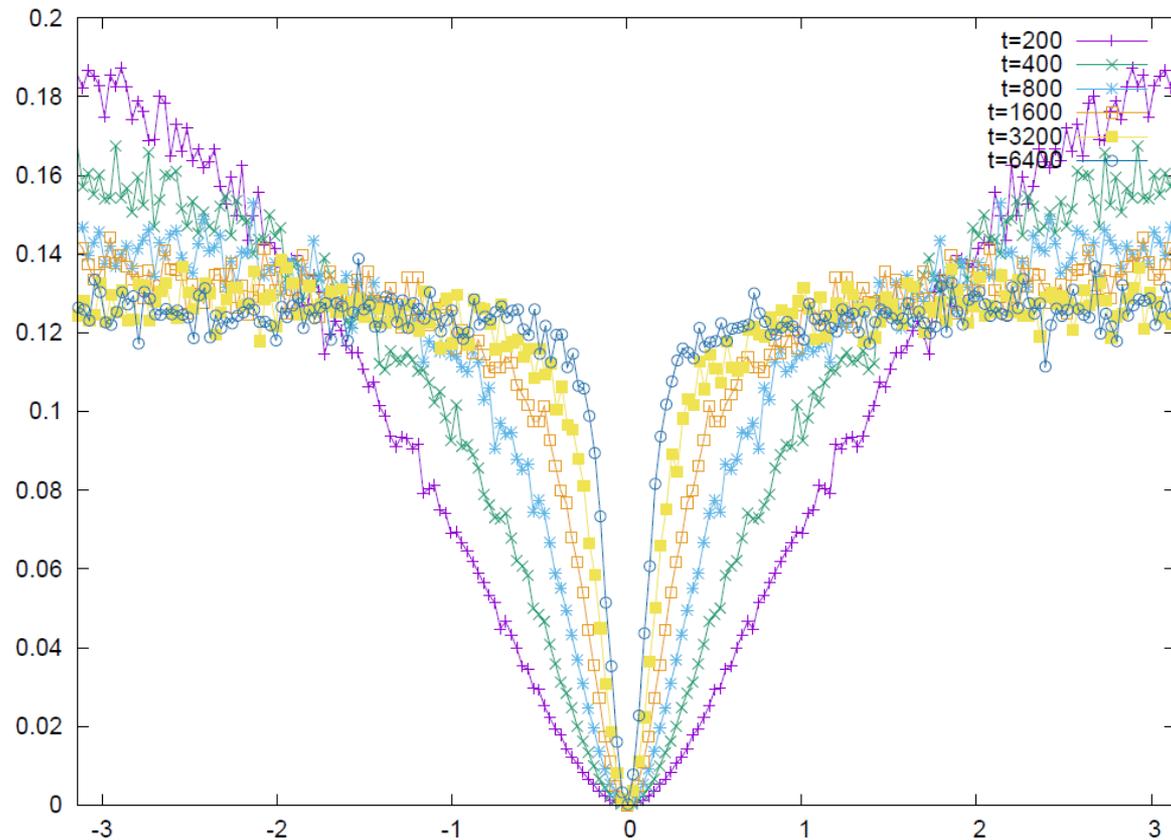
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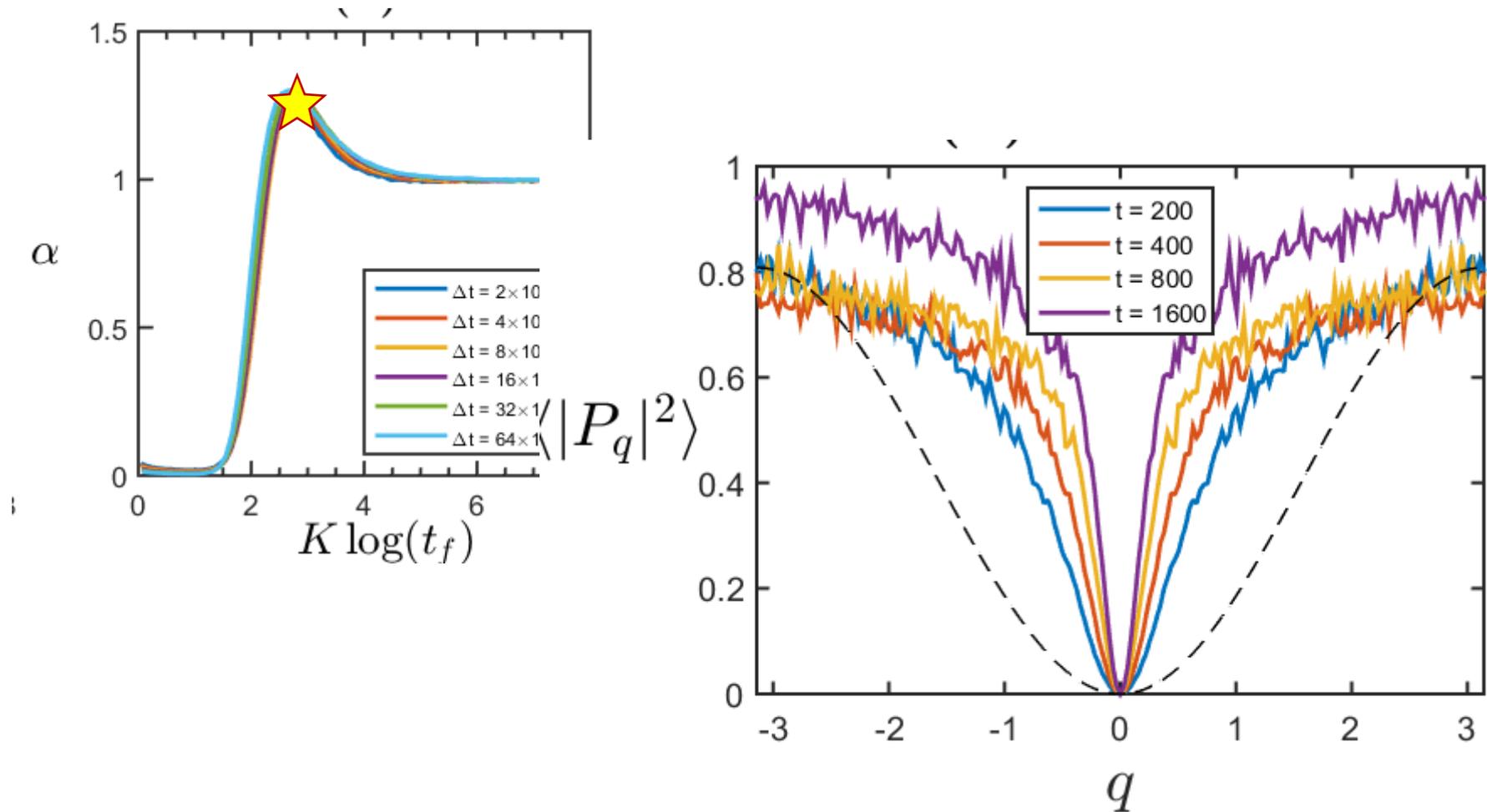


# Extra slides

# Prethermalization (longer times)



# Super-diffusion



# Finite size effect

